

# LOCAL LIMITS AND TRIPLEABILITY

TIM BROOK

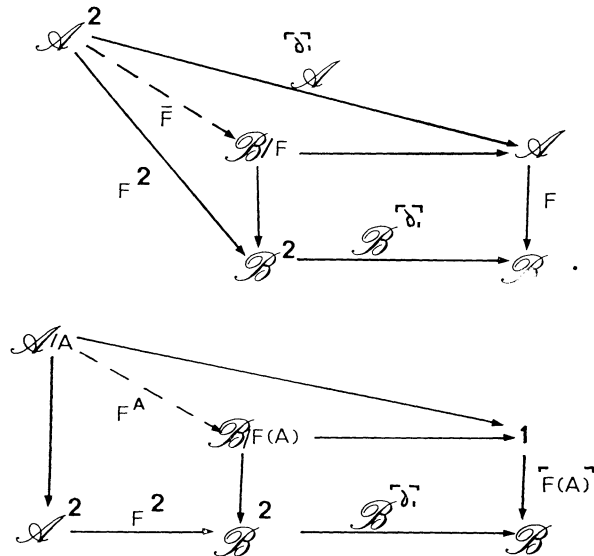
If  $A$  is an object in a category  $\mathcal{A}$ , the properties of  $\mathcal{A}/A$  (the category of objects over  $A$ ) may be considered as local properties of  $\mathcal{A}$ . Using 'local' in this sense, the notion of local universality is defined and some of its basic properties developed. These ideas are then applied in a brief discussion of local adjunction and local limits. Finally two local tripleability theorems are given.

The Lawvere comma category of the diagram

$$1_{\mathcal{B}}: \mathcal{B} \longrightarrow \mathcal{B} \longleftarrow \mathcal{A}: F$$

is denoted by  $\mathcal{B}/F$ , in particular  $\mathcal{B}/B$  denotes the category of objects over  $B$ , when  $B$  is an object of  $\mathcal{B}$ .

Given a functor  $F: \mathcal{A} \rightarrow \mathcal{B}$  we define [3]  $\bar{F}: \mathcal{A}^2 \rightarrow \mathcal{B}/F$  and, for each object  $A$  of  $\mathcal{A}$ ,  $F^A: \mathcal{A}/A \rightarrow \mathcal{B}/F(A)$  by the following pull-back diagrams in  $\mathcal{C}\mathcal{A}\mathcal{T}$ :-



LEMMA 1. For any category  $\mathcal{C}$  there are isomorphisms making

$$\begin{array}{ccc} (\mathcal{B}^2)^{\mathcal{C}} & \xrightarrow{\bar{F}^{\mathcal{C}}} & (\mathcal{B}/F)^{\mathcal{C}} \\ \wr & & \wr \\ (\mathcal{B}^{\mathcal{C}})^2 & \xrightarrow{\bar{F}^{\mathcal{C}}} & \mathcal{B}^{\mathcal{C}}/F^{\mathcal{C}} \end{array} \quad \text{commute.}$$