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LOCAL LIMITS AND TRIPLEABILITY

TIM BROOK

If A is an object in a category $\mathscr{A}$, the properties of $\mathscr{A}/A$ (the category of objects over A) may be considered as local properties of $\mathscr{A}$. Using 'local' in this sense, the notion of local universality is defined and some of its basic properties developed. These ideas are then applied in a brief discussion of local adjunction and local limits. Finally two local tripleability theorems are given.

The Lawvere comma category of the diagram

$$1_{\mathscr{B}}\colon \mathscr{B} \longrightarrow \mathscr{B} \longleftarrow \mathscr{A}\colon F$$

is denoted by $\mathscr{B}/F$, in particular $\mathscr{B}/B$ denotes the category of objects over B, when B is an object of $\mathscr{B}$.

Given a functor $F\colon \mathscr{A} \to \mathscr{B}$ we define [3] $\bar{F}\colon \mathscr{A}^2 \to \mathscr{B}/F$ and, for each object A of $\mathscr{A}$, $F^A\colon \mathscr{A}/A \to \mathscr{B}/F(A)$ by the following pullback diagrams in $\mathscr{CAT}$:—

$$\begin{tikzcd}
\mathscr{A}^2 \arrow[dr, dashed, "\bar{F}"] \arrow[drr, "\ulcorner\delta_1\urcorner_{\mathscr{A}}"] \arrow[ddr, "F^2"'] & & \\
& \mathscr{B}/F \arrow[r] \arrow[d] & \mathscr{A} \arrow[d, "F"] \\
& \mathscr{B}^2 \arrow[r, "\ulcorner\delta_1\urcorner_{\mathscr{B}}"] & \mathscr{B}
\end{tikzcd} .$$

$$\begin{tikzcd}
\mathscr{A}/A \arrow[dr, dashed, "F^A"] \arrow[drr] \arrow[dd] & & \\
& \mathscr{B}/F(A) \arrow[r] \arrow[d] & 1 \arrow[d, "\ulcorner F(A)\urcorner"] \\
\mathscr{A}^2 \arrow[r, "F^2"] & \mathscr{B}^2 \arrow[r, "\ulcorner\delta_1\urcorner_{\mathscr{B}}"] & \mathscr{B}
\end{tikzcd}$$

LEMMA 1. *For any category $\mathscr{C}$ there are isomorphisms making*

$$\begin{tikzcd}
(\mathscr{B}^2)^{\mathscr{C}} \arrow[r, "\bar{F}^{\mathscr{C}}"] \arrow[d, "\wr\wr"] & (\mathscr{B}/F)^{\mathscr{C}} \arrow[d, "\wr\wr"] \\
(\mathscr{B}^{\mathscr{C}})^2 \arrow[r, "\overline{F^{\mathscr{C}}}"] & \mathscr{B}^{\mathscr{C}}/F^{\mathscr{C}}
\end{tikzcd}$$

commute.