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REPRESENTATIONS OF B^*-ALGEBRAS ON BANACH SPACES

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This paper deals with continuous irreducible representations of a B^*-algebra on a Banach space. The main result is that if π is a continuous irreducible representation of a B^*-algebra A on a reflexive Banach space X, and if there is a subset S of A such that the intersection of the null spaces of the operators $\pi(a)$ for all $a \in S$ is a nonzero, finite dimensional subspace of X, then X is a Hilbert space in an equivalent norm and π is similar to a $*$-representation of A on this Hilbert space.

In [7], R. V. Kadison raised the question of whether every continuous representation of a B^*-algebra on a Hilbert space is similar to a $*$-representation. Recently, J. Bunce, in [4], answered Kadison's question affirmatively for a class of B^*-algebras which includes the GCR algebras of Kaplansky. Also, the present author proved an affirmative result concerning this question in [2] (a new proof of this result is given in § 2; see Corollary 2.3). However, the general question remains open. In this paper we consider this question of Kadison in the special case where the representation is assumed to be irreducible. Actually, the problem we consider (in the irreducible case) is more general than Kadison's problem, since we allow the representation space to be a Banach space X. Then we prove under certain conditions on the representation that X is a Hilbert space in an equivalent norm, and that the given representation of the B^*-algebra is similar to a $*$-representation of the algebra on this Hilbert space. A precise statement of our main result is in the abstract above. It is an open question whether the existence of a continuous irreducible representation of a B^*-algebra on a Banach space X necessitates that X is a Hilbert space in an equivalent norm.

At this point we introduce some notation and terminology. *Throughout this paper X is a Banach space and A is a B^*-algebra.* All norms, except for particular norms introduced in context, are denoted by $\|\cdot\|$. The normed dual of X is denoted by X^*. If $x \in X$ and $\alpha \in X^*$, we often use the notation $\langle x, \alpha\rangle$ for $\alpha(x)$. $\mathscr{B}(X)$ is the algebra of all bounded linear operators on X. If $T \in \mathscr{B}(X)$, then $\mathscr{R}(T)$ and $\mathscr{N}(T)$ denote the range and null space of T, respectively.

A nonzero subalgebra B of $\mathscr{B}(X)$ is irreducible (or acts irreducibly) on X if the only closed B-invariant subspaces of X are $\{0\}$ and X. B is strictly irreducible if the only B-invariant subspaces of X are $\{0\}$ and X. If π is a nonzero representation of A into $\mathscr{B}(X)$,