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SIERPINSKI CURVES IN FINITE 2-COMPLEXES

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In this note certain one-dimensional continua are defined for finite 2-complexes. These continua, called S-curves, are a generalization of the Sierpinski plane universal curve. By a 2-complex is meant a finite connected 2-dimensional euclidean polyhedron which has a triangulation such that every 1-simplex is the face of at least one 2-simplex. It is shown that any two S-curves in a 2-complex are homeomorphic. In addition, it is established that two 2-complexes (with the property that every 1-simplex in a triangulation is the face of two or more 2-simplexes) are homeomorphic if and only if the corresponding S-curves are homeomorphic.

In 1916 Sierpinski [4] described a one-dimensional continuum that is known as the Sierpinski plane universal curve. In 1958 Whyburn [7] defined the notion of an S-curve in a 2-sphere and established that an S-curve in a 2-sphere is homeomorphic to the Sierpinski plane universal curve. In 1966 Borsuk [1] defined an S-curve in a surface. He established that any two S-curves in a given surface are homeomorphic and that two surfaces are homeomorphic if and only if the corresponding S-curves are homeomorphic. In this paper the same type of theorems are established for certain 2-complexes.

In order to define an S-curve in a 2-complex, it is necessary to introduce some terminology from Whittlesey [5] or [6]. A point x in a 2-complex K is a *regular point* if it has a neighborhood in K homeomorphic to the plane (euclidean 2-dimensional space). The *regular part* of K is the collection of all regular points in K. The points of K which are not regular are called *singular*; the collection of all singular points in K constitute the *singular graph* of K. Let $D_1, D_2, \cdots$ be a sequence of mutually disjoint closed discs contained in the regular part of K. Then $\mathrm{A}(K) = K - \bigcup_{i=1}^{\infty} \operatorname{Int} D_i$ (Int = interior in the sense of manifolds) is said to be an *S-curve in K* provided that $\bigcup_{i=1}^{\infty} D_i$ is dense in K and the diameters of the D_i converge to zero. Note that if the 2-complex is also a surface, then this definition is precisely that of Borsuk [1, pp. 81–82].

LEMMA 1. *Let K be a 2-complex and Σ an upper semi-continuous decomposition of K with the property that every nondegenerate element of Σ is contained in the regular part of K and each nondegenerate element has arbitrarily small neighborhoods (in K) homeomorphic with the plane. Then the decomposition space K_Σ is homeomorphic to K.*