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# FOURIER TRANSFORMS OF ODD AND EVEN TEMPERED DISTRIBUTIONS

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**In this paper certain previous results of the author concerning Abelian theorems for the Fourier transform of distributions are generalized to two new distribution spaces, those of odd and even tempered distributions. These spaces arise in the consideration of Fourier sine and cosine transforms of distributions.**

In [2] Abelian theorems concerning the Fourier transform of functions provided the initial motivation for similar results about the Fourier transform of distributions. They also contributed directly to these results through the representability of certain types of distributions by functions. It turns out that the analogous procedure is possible with Fourier sine and cosine transforms, leading to the space of even tempered distributions ($\mathscr{S}_e'$) and the space of odd tempered distributions ($\mathscr{S}_0'$).

The basic idea is to generalize the facts for the classical transform that the Fourier transform of an even function is actually a cosine transform and the Fourier transform of an odd function is actually a sine transform. Then as in [2] Abelian theorems can be obtained for these transforms of distributions which are representable in certain ways by functions. In § 5, results of this type are obtained for semiregular distributions, those which are regular over a subset of their respective supports.

With this approach, classical results for both the Fourier sine transform and Fourier cosine transform yield distributional results for these two transforms and can then be combined to yield results about the Fourier transform of a distribution itself. Thus, not only do we have a direct generalization of results for Fourier sine and cosine transforms of functions and hence an alternate approach to Abelian theorems about the Fourier transform, but we are dealing with the larger distribution spaces, $\mathscr{S}_e'$ and $\mathscr{S}_0'$.

2. **Notation and definitions.** The evaluation of a distribution $T$ at a test function $\varphi$ will be denoted by $\langle T, \varphi\rangle$. All integrals are Lebesgue integrals and $f \in BV(\Omega)$ will mean the function $f$ is of bounded variation over the set $\Omega$. $x \rightarrow \pm a$ is shorthand for the two statements $x \rightarrow a^-$ (approach from the left only) and $x \rightarrow a^+$ (approach from the right only). As usual, $f \sim Kg$ $(x \rightarrow a)$ for $K \neq 0$ will mean $f/g \rightarrow K$ as $x \rightarrow a$. If at any time the variable in a given expression is not