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CONDITIONS UNDER WHICH A CONNECTED REPRESENTABLE SPACE IS LOCALLY CONNECTED

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In this paper it is shown that every strongly locally homogeneous Hausdorff continuum is locally connected. It is known that every connected representable space is homogeneous and that every locally connected representable space is strongly locally homogeneous; this paper investigates the problem of whether or not every connected representable space is locally connected.

The discovery that homogeneity did not characterize the circle among planar continua motivated other classifications of continua in terms of the actions of their homeomorphism groups, and the pseudo-arc remains the most tortuous proving grounds for testing such topological properties. Those homogeneity-like properties that are not possessed by the pseudo-arc appear to be closely related to local connectedness. Such properties include strong local homogeneity, representability, (strong) 2-homogeneity and countable dense homogeneity.

In this paper it is shown that every strongly locally homogeneous Hausdorff continuum is locally connected. It is known that every connected representable space is homogeneous and that every locally connected representable space is strongly locally homogeneous; this paper investigates the problem of whether or not every connected representable space is locally connected. It follows from results of Ben Fitzpatrick, Jr. and Ralph Bennett that every connected locally compact separable metric representable space is locally connected. Arguments of Fitzpatrick and de Groot are used to show that if (X, τ) is a representable connected complete metric space such that τ does not have a base of totally disconnected sets, then (X, τ) is locally connected (and hence arcwise connected and strongly locally homogeneous). The authors do not know of even a homogeneous connected complete metric space with a base of totally disconnected sets; however, there exists a separable connected metric topology that has such a base.

Let (X, τ) be a topological space. We let $H(X)$ denote the group of all homeomorphisms from the space (X, τ) onto itself and let i denote the identity of $H(X)$. If $A \subset X$, then $A' = \{h \in H(X): h \mid A = i \mid A\}$ and if G is a subgroup of $H(X)$, then $G' = \{x \in X: g(x) = x \text{ for each } g \in G\}$. If $G \subset H(X)$ and $A \subset X$, then $G(A) = \{g(a): g \in G \text{ and } a \in A\}$. We write $G(x)$ in room of $G(\{x\})$. Throughout this paper all spaces are assumed to be Hausdorff.