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COMMUTANTS OF SOME QUASI-HAUSDORFF MATRICES

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Let $B(c)$ denote the Banach algebra of bounded linear operators over c, the space of convergent sequences, and Γ^* the subalgebra of conservative infinite matrices. Given an upper triangular matrix A in Γ^*, a sufficient condition is established for the commutant of A in Γ^* to be upper triangular. Also determined is the commutant, in $B(c)$, of certain quasi-Hausdorff matrices.

The spaces of bounded, convergent and null sequences will be denoted by m, c, c_0 respectively, and l will denote the set of sequences x satisfying $\sum_k |x_k| < \infty$. Let Δ^* denote the algebra of conservative upper triangular matrices; i.e., $A \in \Delta^*$ implies $A: c \to c$ and $a_{nk} = 0$ for $n > k$. $\mathscr{H}^*$ will denote the algebra of conservative quasi-Hausdorff transformations, and Γ the algebra of all conservative matrices. Γ_a^{1*} is the quasi-Hausdorff transformation generated by $\mu_n = a(n+a)^{-1}$, $a > 1$. For other specialized terminology the reader can consult [3] or [5].

One cannot answer commutant questions for upper or lower triangular matrices in $B(c)$ by taking transposes. For example, let C denote the Cesàro matrix of order 1. C^T is not conservative. On the other hand, the matrix $A = (a_{nk})$ defined by

$$a_{nk} = \begin{cases} 1 \text{ for } n = \binom{j+1}{2},\ \binom{j}{2} + 1 \leqq k \leqq n\,; \qquad j = 1, 2, \cdots, \\ 0 \text{ otherwise}, \end{cases}$$

is conservative, but A^T is not. It is true that the transpose of any conservative quasi-Hausdorff matrix is a conservative Hausdorff matrix. C shows that the converse is false.

We begin with some results analogous to those of [3] and [5].

THEOREM 1. *Let $A \in \Delta^*$. If A has the property that*
(1) *for each $t \in m$, $n \geqq 0$, $(A - a_{nn}I)t = 0$ implies $t \in$ linear span $\{e^0, e^1, \cdots, e^n\}$, then every matrix B with finite norm which commutes with A is upper triangular.*

$B \leftrightarrow A$ *implies*

$$(2) \qquad \sum_{j=0}^{k} b_{nj}a_{jk} = \sum_{j=n}^{\infty} a_{nj}b_{jk}\,; \qquad n, k = 0, 1, 2, \cdots.$$

Set $k = 0$ to get

$$b_{n0}a_{00} = \sum_{j=n}^{\infty} a_{nj}b_{j0}\,; \qquad n = 0, 1, 2, \cdots,$$