PACIFIC JOURNAL OF MATHEMATICS
Vol. 54, No. 2, 1974

# RIGHT CONGRUENCES AND SEMISIMPLICITY FOR REES MATRIX SEMIGROUPS

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**In this paper a comparison is made for various definitions of radicals and semisimplicity on the class of Rees matrix semigroups. Preliminary to this, results are obtained on various types of right congruences. This is equivalent to characterizing various types of automata having a Rees matrix semigroup as an input semigroup.**

Let $K$ and $L$ be arbitrary sets and $G$ a group. The group $G$ with a zero adjoined is denoted by $G^0$. We let $\phi$ be a mapping of $L \times K$ into $G^0$ and define a product on $K \times G^0 \times L$ by

$$(1) \qquad (e, g, f)(u, w, v) = (e, g\phi(f, u)w, v).$$

If we identify all $(e, 0, f)$ by $\theta$ we obtain a semigroup with zero $\theta$ called a Rees matrix semigroup with zero [**1**]. If we restrict our attention to $K \times G \times L$ and restrict $\phi$ to be into $G$ then the product in (1) defines a Rees matrix semigroup without zero [**1**]. These semigroups have played an extremely important role in the characterization of simple semigroups.

Recently, several attempts have been made to obtain a structure theory for semigroups by first defining a radical and, subsequently, semisimplicity. In order to bring the two approaches somewhat closer together we examine the various definitions of semisimplicity as they apply to Rees matrix semigroups. Preliminary to this we characterize modular, maximal modular, and transitive right congruences, as well as the general right congruences, for Rees matrix semigroups.

**1. Right congruences of a Rees matrix semigroup.** As above, we describe our Rees matrix semigroup with zero using the notation $S = (K \times G \times L) \cup \{\theta\}$ and $(e, 0, f) = \theta$ for all $e \in K$ and $f \in L$. Define $L_0 = \{f: f \in L \text{ and } \forall\, k \in K, \phi(f, k) = 0\}$, $L_1 = L - L_0$ and $f(K) = \{k : k \in K \text{ and } \phi(f, k) \neq 0\}$. Let

(2) $U$ be a set not containing 1 or 0 and $\{K_i : i \in U\} \cup \{K_0\}$ be a decomposition of $K$ with $K_0$ possibly empty. To each $K_i$ with $i \in U$ assign a subgroup $H_i$ of $G$; to $K_0$ assign $G^0$.

(3) $\alpha : K \to G$.