PACIFIC JOURNAL OF MATHEMATICS
Vol. 55, No. 2, 1974

ON SOME GROUP ALGEBRA MODULES RELATED TO WIENER'S ALGEBRA M_1

TENG-SUN LIU, ARNOUD VAN ROOIJ AND JU-KWEI WANG

Along with his study of the general Tauberian theorem in L_1, N. Wiener introduced the algebra M_1 which consists of all those continuous functions f on the real line $\boldsymbol{R}$ for which

$$\sum_{n=-\infty}^{\infty} \max_{x \in [n, n+1]} |f(x)| < \infty .$$

He proved that many features of L_1, including the general Tauberian theorem, are shared by M_1. In this paper to generalize M_1 to an arbitrary locally compact group G. While doing this, a host of $L_1(G)$-modules mutually related by conjugation and the operation of forming multiplier modules. $\mathscr{M}_1(G)$ is among them. In case G is abelian, $\mathscr{M}_1(G)$ is a Segal algebra, so that it has the same ideal-theoretical structure as $L_1(G)$. If further $G = \boldsymbol{R}$, $\mathscr{M}_1(G)$ reduces to the Wiener algebra M_1 with an equivalent norm.

1. Our notations are basically the same as those used in [3]. We use, however, $\boldsymbol{C}$ to denote the complex number field. Throughout the paper, G is a locally compact group with a left Haar measure λ. Instead of $C_{00}(G)$, $L_p(G)$ etc. we write C_{00}, L_p etc. We view L_1 as a subspace of M. We identify two functions that are equal almost everywhere.

For a function f on G define f' by

$$f'(x) = f(x^{-1})\Delta(x^{-1}) ,$$

where Δ denotes the modular function of G. Then $f'' = f$ and $(f * g)' = g' * f'$ for $f, g \in L_1$.

If B is a left Banach module over L_1 (see [3; 32.14]), then B^* becomes a left Banach module by

$$(j, f * \phi) = (f' * j, \phi) \qquad (j \in B; \phi \in B^*; f \in L_1) .$$

If $B = L_p (1 \leqq p < \infty)$ or $B = C_0$ the module operation on B^* coincides with the convolution operation on $L_q (q = p/(p-1))$ or M.

Let B be a left Banach module over L_1. By [3; 32.22], $\{f * j : f \in L_1; j \in B\}$ is a closed submodule of B. We denote this submodule by $L_1 * B$ or B_{abs}. We call B *absolutely continuous* if $B_{abs} = B$.

Suppose B is a Banach space, and there is a map $(j, x) \mapsto j_x$ of $B \times G$ into B such that

(1) $j_1 = j (j \in B)$,