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TWO THEOREMS ON GROUPS OF CHARACTERISTIC 2-TYPE

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D. Gorenstein has made the following conjecture: suppose that G is a finite simple group which is simultaneously of characteristic 2-type and characteristic 3-type. Then G is isomorphic to one of $PSp(4, 3)$, $G_2(3)$ or $U_4(3)$. In this paper, we prove two results which, taken together, yield a proof of this conjecture under the additional assumption that G has 2-local 3-rank at least 2.

1. Introduction. In this paper we study finite simple groups, all of whose 2-local and 3-local subgroups are 2-constrained and 3-constrained respectively. The results we obtain are extensions of Thompson's theorem ES, and their relation to simple groups of characteristic 2-type is entirely analogous to the relation of theorem ES to simple N-groups.

The two Main Theorems are actually slight extensions of a conjecture of Gorenstein [10], and we refer the reader to [10] for a more detailed discussion of these ideas.

It will be convenient, before stating our main results, to develop some notation, most of which is standard.

Let X be a group, Y a subgroup of X, and π a set of primes. Then $\mathscr{N}_X(Y;\pi)$ denotes the set of Y-invariant π-subgroups of X. In particular, if the only Y-invariant π-subgroup of X is 1, we write $\mathscr{N}_X(Y;\pi) = \{1\}$.

For a finite group X, $\pi(X)$ is the set of prime divisors of $|X|$. As in [26], the subdivision of $\pi(X)$ into π_1, π_2, π_3 and π_4 will be important. We recall that $p \in \pi_3 \cup \pi_4$ if a S_p-subgroup P of G has a normal abelian subgroup of rank at least 3, which we write as $SCN_3(P) \neq \emptyset$. Moreover,

$$p \in \pi_3 \text{ if } SCN_3(P) \neq \emptyset \text{ and } \mathscr{N}_X(P;\, p') \neq \{1\}$$
$$p \in \pi_4 \text{ if } SCN_4(P) \neq \emptyset \text{ and } \mathscr{N}_X(P;\, p') = \{1\}\,.$$

If p is a prime, X a group, and P a S_p-subgroup of $O_{p',p}(X)$, we say that X is p-constrained if $C_X(P) \leqq O_{p',p}(X)$.

For p a prime and X a group, a p-local subgroup of X is the normalizer of some nonidentity p-subgroup of X.

We say that X is of characteristic p-type if $p \in \pi_4$ and every p-local subgroup of X is p-constrained.

With these definitions we can now state Gorenstein's conjecture: Suppose that G is a finite simple group, p an odd prime, and