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ON PERTURBATION OF DIFFERENTIAL OPERATORS

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The theory of spectral operators, when applied to eigenfunction expansions, covers the unconditionally convergent case. However, by perturbing certain spectral differential operators, J. Schwartz has obtained differential operators which are not spectral but whose eigenfunctions span the whole space. In this paper we show how new norms can be constructed so as to make these perturbed differential operators spectral. This we achieve by showing that such operators have an underlying generalized spectral measure (as defined by V. È. Ljance) and that every generalized spectral measure is essentially a C-spectral measure.

It may be shown that the topology defined by the new norm is finer than the outer spectral topology introduced by Ljance. Thus the theory of C-spectral measures both generalizes the concept as well as sharpens some of the results of the theory of generalized spectral measures. In this connection, it will be noted that the norm constructed by Smart is the inner spectral norm of the theory of C-spectral measures.

We denote by $\bar{X}$ a Banach space over the field C of all complex numbers; the norm of $\bar{X}$ is denoted by $\|\cdot\|$. The set of all bounded linear operators on $\bar{X}$ into itself is denoted $B(\bar{X})$. For any vector space X, we denote by $L(X)$ the set of all linear transformations on X into itself. The adjoint space of $\bar{X}$ is denoted $\bar{X}^*$. If $T \in B(\bar{X})$, we denote by $N(T)$ the kernel of T and by T^* the adjoint of T. B_0 will denote the σ-algebra of all Borel subsets of the complex plane Λ.

1. Spectral measures and their generalizations.

DEFINITION 1 (Ljance). A subring A of B_0 is said to be admissible if and only if it contains along with each of its members every Borel subset of that member.

If an admissible subring is also an algebra then it equals the whole of B_0, which is obviously an admissible subring of itself.

DEFINITION 2. Let X be a dense linear subspace of $\bar{X}$ and A an admissible subring of B_0. Let $E: A \to L(X)$ be a mapping such that

D(i) $E(\delta)E(\sigma) = E(\delta \cap \sigma)$ $(\delta, \sigma \in A)$;

D(ii) For each $x \in X$, the mapping $E(\cdot)x: A \to \bar{X}$ is countably