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A MAP OF E^3 ONTO E^3 TAKING NO DISK ONTO A DISK

EDYTHE P. WOODRUFF

An example is given of an u.s.c. decomposition in which no disk in E^3 maps onto a disk under the natural projection map P, and, furthermore, the decomposition space E^3/G is homeomorphic to E^3. Each nondegenerate element is a tame arc. The image $P(H)$ of the set of nondegenerate elements is 0-dimensional, although $\mathrm{Cl}\, P(H)$ is E^3. The basic construction used is called a knit Cantor set of nondegenerate elements.

Bing and Borsuk **[3]** have given an example of a 3-dimensional absolute retract R containing no disk. They define a particular u.s.c. decomposition of E^3 that yields R as the decomposition space. Hence, their example is a closed map of E^3 taking no disk onto a disk, but, of course, their image is not E^3.

In **[8]** the author defined a set $X \subset E^3$ to be the P-lift of a set Y contained in the decomposition space E^3/G if and only if X and Y are homeomorphic and the image of X under the natural projection is Y. A disk is said to said to be P-liftable if and only if it has a P-lift. Using this terminology, the example that is constructed in this note has no P-liftable disk in the image space.

In **[1]** Armentrout asked whether there exists a pointlike decomposition G of E^3 such that there is a 2-sphere S in E^3/G that can not be approximated by a P-liftable sphere. This was first answered by the author in **[9]** by giving an example of a space E^3/G containing such a 2-sphere. In the decomposition space of this note no 2-sphere is P-liftable. Hence, this space is another answer to Armentrout's query.

The construction we describe in this note is based on a knit example in the author's papers **[6]**, **[7]**, and **[9]**. It is assumed that the reader is familiar with this example and the notations in **[6]**. We also need the following definitions.

DEFINITION. Let J_1' be the circle in the x—y plane with radius 1 and center at the origin, and J_2' be the circle in the y—z plane with radius 1 and center at $y = -1$, $z = 0$. Any two tame simple closed curves J_1 and J_2 in E^3 are said to *simply link* if and only if there is a homeomorphism of E^3 onto itself taking J_1 and J_2 onto J_1' and J_2', respectively. Two disjoint compact sets S_1 and S_2 are said to *simply link* if and only if there exist simple closed curves $J_1 \subset S_1$ and $J_2 \subset S_2$ such that J_1 and J_2 simply link.