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MOMENT SEQUENCES IN l^p

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Let $p > 0$. Conditions are derived, each necessary and sufficient, for a moment sequence to be in l^p. It is shown that the moment sequences in l^p are dense in l^p. For $p = 2$, these results were obtained by G. G. Johnson.

G. G. Johnson obtained a necessary and sufficient condition for a moment sequence to be in l^2, and showed that the moment sequences in l^2 are dense in l^2. This paper shows that the same conclusions hold in any l^p space. The proofs are similar to and improvements of those in G. G. Johnson, Pacific J. Math., **46**(1973), 201–207.

LEMMA 1. *Let* $0 < p < 1$, $q > 0$. *If* $a_n = 1 - (n+1)^{-p}$, *then* $\{a_n^n\} \in l^q$.

Proof. $a_n^{nq} = \exp(qn \log(1 - (n+1)^{-p})) < \exp(qn(-(n+1)^{-p})) = (\exp(qn(n+1)^{-p}))^{-1} < [\Sigma_{k=0}^{N} (qn(n+1)^{-p})^k / k!]^{-1}$, where N satisfies $N(1-p) > 1$. Then

$$\sum_{n=1}^{\infty} a_n^{nq} < \sum_{n=1}^{\infty} [(qn(n+1)^{-p})^N / N!]^{-1} = N!\, q^{-N} \sum_{n=1}^{\infty} [(n+1)^p / n]^N,$$

which converges if and only if $\Sigma_{n=1}^{\infty} n^{-(1-p)N}$ converges, and the latter is a convergent p-series.

THEOREM 1. *Let* $p > 0, f \in BV[0,1], \mu_n = \int_0^1 t^n df$. *For each* $\{a_n\}$ *such that* $0 \leqq a_n < 1$, *and* $\{a_n^n\} \in l^p$, *the following are equivalent.*

(i) $\{\mu_n\} \in l^p$

(ii) $\left\{ f(1) - (1 - a_n^n)^{-1} \int_{a_n}^1 f(t) dt^n \right\}_{n=1}^{\infty} \in l^p$.

Lemma 1 shows such $\{a_n^n\}$ exist.

Proof. Split the integral for μ_n at a_n and integrate by parts to obtain, as in **[1]**, $\mu_n = a_n^n(\delta_n - \gamma_n) + (f(1) - \delta_n)$, where $\delta_n = (1 - a_n^n)^{-1} \int_{a_n}^1 f(t)\, dt^n$ and $\gamma_n = (a_n^n)^{-1} \int_0^{a_n} f(t) dt^n$. Since $|\delta_n - \gamma_n|$ is bounded, $\{a_n^n(\delta_n - \gamma_n)\} \in l^p$, so that $\{\mu_n\} \in l^p$ if and only if $\{f(1) - \delta_n\}_{n=1}^{\infty} \in l^p$.