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ON M-PROJECTIVE AND M-INJECTIVE MODULES

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In this paper necessary and sufficient conditions are obtained for a direct sum $\bigoplus_{\alpha \in J} A_\alpha$ of R-modules to be M-injective in the sense of Azumaya. Using this result it is shown that if $\{A_\alpha\}_{\alpha \in J}$ is a family of R-modules with the property that $\bigoplus_{\alpha \in K} A_\alpha$ is M-injective for every countable subset K of J then $\bigoplus_{\alpha \in J} A_\alpha$ is itself M-injective. Also we prove that arbitrary direct sums of M-injective modules are M-injective if and only if M is locally noetherian, in the sense that every cyclic submodule of M is noetherian. We also obtain some structure theorems about Z-projective modules in the sense of Azumaya, where Z denotes the ring of integers. Writing any abelian group A as $D \oplus H$ with D divisible and H reduced, we show that if A is Z-projective then H is torsion free and every pure subgroup of finite rank of H is a free direct summand of H.

Most of these results were motivated by the results of B. Sarath and K. Varadarajan regarding injectivity of direct sums.

1. *M-projective and M-injective modules.* Throughout this paper R denotes a ring with $1 \neq 0$ and all the modules considered are left unitary modules over R. By an ideal in R we mean a left ideal in R. M denotes a fixed R-module. We first recall the notions of M-projective and M-injective modules originally introduced by one of the authors [1].

DEFINITION 1.1. An R-module H is called M-projective, if given a diagram

$$\begin{array}{ccccc} & & H & & \\ & & \downarrow{\scriptstyle f} & & \\ M & \xrightarrow{\varphi} & N & \longrightarrow & 0 \end{array}$$

of maps of R-modules with the horizontal sequence extact, $\exists$ a map $h: H \rightarrow M$ such that $\varphi \circ h = f$.

The notion of an M-injective module is defined dually.

REMARK 1.2. Regarding R as a left module over itself in the usual way it turns out that R-injective modules are the same as the injective modules over R. However R-projective modules are not the same as projective modules over R.