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NORMS OF RANDOM MATRICES

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Under rather general conditions on the matrix entries, we obtain estimates for the probability distribution of the norm of a random matrix transformation from ℓ_n^2 to ℓ_m^q $(2 \leqq q < \infty)$. Asymptotically, the expected norm is remarkably small and this enables us to produce an interesting class of bounded linear operators from ℓ^2 to ℓ^q. As an application, we complete the characterization of (p, q)-absolutely summing operators on Hilbert space, thereby answering a question left open by several previous authors.

1. **Introduction.** Many questions in the theory of ℓ^p spaces require, for their solution, the existence of finite matrices with ± 1 entries whose norms satisfy prescribed conditions. In several cases the required matrices have been given explicitly: the simplest examples stem from the orthogonality of the Walsh functions (see, for example, [10] where non-complemented subspaces of ℓ^p are constructed) or from the Rademacher functions via Khintchine's inequality (see, for example, [6] where the p-absolutely summing operators on Hilbert space are characterized). In many problems, however, the construction of suitable matrices leads to formidable combinatorial difficulties. We consider in this paper one such problem for which no constructive method is available. The appropriate matrix is obtained here probabilistically by showing that "most" matrices satisfy the prescribed norm inequalities.

Specifically, the problem we consider is that of characterizing the ideal, $\prod_{p,q}$, of (p, q)-absolutely summing operators on Hilbert space. Recall that a bounded linear operator T on ℓ^2 is (p, q)-*absolutely summing* $(1 \leqq q \leqq p \leqq \infty)$ if $(\|Tx_n\|)_{n=1}^{\infty} \in \ell^p$ whenever $(x_n)_{n=1}^{\infty}$ is a sequence of elements of ℓ^2 with the property that $(\langle x_n, y\rangle)_{n=1}^{\infty} \in \ell^q$ for each $y \in \ell^2$. This problem has received a good deal of attention in recent years and the known results are described below. We denote by $\mathfrak{S}_r (1 \leqq r < \infty)$ the *Schatten r-class* of all compact linear operators T on ℓ^2 for which $\sum_{n=1}^{\infty} |\lambda_n|^r < \infty$ where $\{\lambda_n\}_{n=1}^{\infty}$ are the eigenvalues of $(T^*T)^{1/2}$, counted according to multiplicities (and arranged in order of decreasing modulus); for convenience the class of *all* bounded linear operators on ℓ^2 is denoted by $\mathfrak{S}_\infty$.

We then have:

(a) *if* $p = q < \infty$, $\Pi_{p,q} = \mathfrak{S}_2$;
(b) *if* $p = \infty$ *or* $(1/q) - (1/p) \geqq 1/2$, $\Pi_{p,q} = \mathfrak{S}_\infty$;