PACIFIC JOURNAL OF MATHEMATICS
Vol. 60, No. 2, 1975

EMBEDDINGS OF SHAPE CLASSES OF COMPACTA IN THE TRIVIAL RANGE

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We show that for compacta X, $Y \subset R^n$, $n \geqq 5$, satisfying the small loops condition and having dimensions in the trivial range with respect to n, $Sh(X) = Sh(Y)$ if and only if $R^n - X \approx R^n - Y$. As a corollary we obtain the following result whose statement is void of shape: If X, $Y \subset R^n$, $n \geqq 5$, are homeomorphic compacta satisfying the small loops condition and having dimensions in the trivial range with respect to n, then $R^n - X \approx R^n - Y$.

1. Main results and introduction. In this paper we are concerned with the general problem of classifying the collection C_Z of compacta in a space Z for which the following property holds: $Sh(X) = Sh(Y)$ is equivalent to $Z - X \approx Z - Y$ ($\approx$ means "is homeomorphic to") for all X, $Y \in C_Z$. Our results apply to compacta in R^n whose dimensions are in the trivial range with respect to n. After defining a fundamental homotopy condition and stating our main results, we will discuss some related work.

Let X be a compactum in a manifold M. We say that X satisfies the *small loops condition* (SLC) if for any neighborhood U of X, there is a neighborhood V of X in U and an $\epsilon > 0$ such that each map of S^1 into $V - X$ of diameter less than ϵ is null homotopic in $U - X$. We say that k is in the *trivial range* with respect to (w.r.t.) n if $2k + 2 \leqq n$ (or equivalently $k \leqq [n/2] - 1$).

THEOREM 1. *Let X, $Y \subset R^n$, $n \geqq 5$, be compacta satisfying* SLC *whose dimensions are in the trivial range* w.r.t. n. *Then, $Sh(X) = Sh(Y)$ if and only if $R^n - X \approx R^n - Y$.*

Theorem 1 generalizes the main result of **[6]** which we recapture in the following corollary. (The paper **[6]** improved upon **[4]** which was the first trivial range work of the nature of Theorem 1.) The 1–ULC hypothesis of Geoghegan and Summerhill is a local condition that traditionally has been used to show that topological embeddings are flat or unknotted, whereas the SLC, the cellularity criterion, and the global 1–alg property are three intimately related (see Proposition 1.5 of **[5]**) global conditions that have traditionally been used to show that topological embeddings have nice complements (are weakly flat) or homeomorphic complements. We will give an example illustrating this point following Corollary 3.