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COARSE UNIFORM CONVERGENCE SPACES

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The coarse uniform convergence space is one which is the coarsest member of its convergence class. Each convergence space is compatible with a coarse uniform convergence structure. The regular topological spaces can be characterized as those whose coarse uniform convergence structures are uniformly regular or, equivalently, as those convergence spaces whose coarse uniform convergence structures are very strongly bounded. Every coarse uniform convergence space has a coarse completion. The coarse uniform convergence spaces which have uniformly regular completions are precisely the coarse uniform spaces.

0. Preliminaries. For any set X, let $F(X)$ be the set of all filters on X. The fixed ultrafilter generated by $\{x\}$ will be denoted by $\dot{x}$. (We shall hereafter use "u.f." as an abbreviation for "ultrafilter".) Let Δ be the diagonal in $X \times X$; we denote by $\dot{\Delta}$ the filter $\{A \subseteqq X \times X: \Delta \subseteqq A\}$.

DEFINITION 0.1. A *uniform convergence structure* I on X is a subset of $F(X \times X)$ which contains $\dot{\Delta}$, is closed under finite intersections, and satisfies the additional conditions;

(a) $\Phi \in I$ implies $\Phi^{-1} \in I$;

(b) $\Phi \in I$ and $\Phi \leqq \Psi$ implies $\Psi \in I$;

(c) If $\Phi, \Psi \in I$ and the composition $\Phi \circ \Psi$ exists, then $\Phi \circ \Psi \in I$.

We will use "u.c.s." as an abbreviation for "uniform convergence structure" or "uniform convergence space", where the latter term refers to a set equipped with a uniform convergence structure. The notion of a u.c.s. was introduced by Cook and Fischer [1], and a reader not already familiar with this notion can refer to [1] for background information.

A filter $\Phi \in F(X \times X)$ is defined to be *Δ-symmetric* if $\Phi = \Phi^{-1}$ and $\dot{\Delta} \geqq \Phi$. The Δ-symmetric members of any u.c.s. I form a base for I. A *Cauchy filter* for a u.c.s. (X, I) is a filter $\mathscr{F} \in F(X)$ such that $\mathscr{F} \times \mathscr{F} \in I$. A u.c.s. is *totally bounded* if every u.f. is Cauchy.

DEFINITION 0.2. A *convergence structure* q on a set X is a function from $F(X)$ into the power set of X satisfying the following conditions:

(a) $x \in q(\dot{x})$, all $x \in X$;

(b) $x \in q(\mathscr{F})$ and $\mathscr{F} \leqq \mathscr{G}$ implies $x \in q(\mathscr{G})$;