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OSCILLATION OF EVEN ORDER DIFFERENTIAL EQUATIONS WITH DEVIATING ARGUMENTS

TSAI-SHENG LIU

The purpose of this paper is to give some oscillation criteria for even order differential equations with deviating arguments.

A continuous real-valued function $f(t)$ which is defined for all large t is called *oscillatory* if it has arbitrary large zero, otherwise it is called *nonoscillatory*.

Our work extends some results obtained by Ladas and Lakshmikantham [3] and Chiou [1] for second order equations.

1. In this section, we are concerned with the equation

$$y^{(n)}(t) - \sum_{j=1}^{m} p_j(t)y(g_j(t)) = 0 \;, \quad n \geqq 2 \text{ an even integer} \;, \tag{1.1}$$

where the following assumptions are assumed to hold:

(I_1) $g_j(t) \leqq t$ on $[a, \infty)$, $j = 1, 2, \cdots, m$ and $g_k(t) < t$ for some $1 \leqq k \leqq m$; $g'_j(t) \geqq 0$ on $[a, \infty)$ and $g_j(t) \to \infty$ as $t \to \infty$, $j = 1, 2, \cdots, m$.

(I_2) $p_j(t) \geqq 0$, $p'_j(t) \leqq 0$ on $[a, \infty)$, $j = 1, 2, \cdots, m$ and $p_k(t) > 0$ on $[a, \infty]$ for the same k as in (I_1).

We shall give a sufficient condition for all bounded solutions of (1.1) to be oscillatory. Our result extends Ladas and Lakshmikatham's Theorems 2.1–2.4 in [3] to arbitrary even order equation (1.1).

LEMMA 1.1 (Lemma 2 in [2]). *If y is a function, which together with its derivatives of order up to $(n-1)$ inclusive, is absolutely continuous and of constant sign on the interval $[a, \infty)$ and $y^{(n)}(t)y(t) \geqq 0$ on $[a, \infty)$, then either*

$$y^{(j)}(t)y(t) \geqq 0 \;, \qquad j = 0, 1, \cdots, n \;,$$

or there is an integer l, $0 \leqq l \leqq n-2$, which is even when n is even and odd when n is odd, such that

$$y^{(j)}(t)y(t) \geqq 0 \;, \qquad j = 0, 1, \cdots, l \;,$$

and

$$(-1)^{n+j}y^{(j)}(t)y(t) \geqq 0 \;, \qquad j = l+1, \cdots, n$$

for t in $[a, \infty)$.