PACIFIC JOURNAL OF MATHEMATICS
Vol. 62, No. 1, 1976

RELATIONS AMONG GENERALIZED MATRIX FUNCTIONS

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Let G be a permutation group of degree m. Let λ be an irreducible, complex character of G. If $A = (a_{ij})$ is an m by m matrix, the generalized matrix function of A corresponding to G and λ is defined by

$$d_\lambda^G(A) = \sum_{\sigma \in G} \lambda(\sigma) \prod_{t=1}^{m} a_{t\sigma(t)}.$$

We obtain relations among generalized matrix functions arising from G and those arising from a subgroup H of G. The methods yield some information about the corresponding symmetry classes of tensors.

Generalized matrix functions were invented by I. Schur to improve E. Fischer's improvement of the Hadamard determinant theorem. Specifically, Schur proved that

$$d_\lambda^G(A) \geqq \lambda(id)\det(A)$$

for all positive semidefinite Hermitian matrices A (write $A \geqq 0$).

1. The main results.

In what has become a classic paper on the subject, S. G. Williamson obtained the following result in **[14]**: *If H is a subgroup of G and if λ is a character of G of degree* 1, *then*

$$d_\lambda^G(A) \leqq [G\colon H] d_\lambda^H(A) \tag{1}$$

for all $A \geqq 0$. In **[6]**, the present author improved Williamson's result as follows: *Let H be a subgroup of G. Let λ be an irreducible character of G. Suppose the restriction of λ to H is $\lambda(id)\chi/\chi(id)$ for some irreducible character χ of H* (i.e., *suppose $\lambda|_H$ is a multiple of χ*). *Then*

$$\lambda(id) d_\lambda^G(A) \leqq [G\colon H]\chi(id) d_\chi^H(A), \tag{2}$$

for every $A \geqq 0$.