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ABELIAN AND NILPOTENT SUBGROUPS OF MAXIMAL ORDER OF GROUPS OF ODD ORDER

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Denote the *maximum* of the orders of all *nilpotent* subgroups A of class at most c, of a finite group G, by $d_c(G)$. Let $A_c(G)$ be the set of all nilpotent subgroups of class at most c and having order $d_o(G)$ in G. Let $A_\infty(G)$ denote the set of all nilpotent subgroups of maximal order of a group G.

The aim of this paper is to investigate the set $A_\infty(G)$ of groups G of odd order and the structure of the groups G with the property $A_2(G) \subseteq A_\infty(G)$. Theorem 1 gives an expression for the number of elements in $A_\infty(G)$. Theorem 2 gives criteria for the nilpotency of groups of odd order.

In this paper G is a finite group, and π is a set of primes. If G is of odd order, then G is solvable **[6]**.

1. Introduction.

Denote the *maximum* of the *orders* of all *nilpotent* subgroups A of class at most c, of a finite group G, by $d_c(G)$. Let $A_c(G)$ be the set of all nilpotent subgroups of class at most c and having order $d_c(G)$ in G. Then $J_c(G)$ is the subgroup of G generated by $A_c(G)$. In particular, $J_1(G) = J(G)$ is the *Thompson subgroup* of G. Moreover, $A_\infty(G)$ is the set of all nilpotnet subgroups of maximal order of a group G. Here $J_\infty(G)$ is the subgroup of G generated by the elements of $A_\infty(G)$.

In this paper G is a finite group, and π is a set of primes. If G is of odd order, then G is solvable **[6]**.

The aim of this paper is to investigate the set $A_\infty(G)$ for groups G of odd order and the structure of the groups G with the property $A_2(G) \subseteq A_\infty(G)$.

We shall give, in Theorem 1, an expression for the number of elements in $A_\infty(G)$. In Theorem 2 we shall state criteria for the nilpotency of groups of odd order.

For groups G with the property $A_2(G) \subseteq A_\infty(G)$, we have the following:

THEOREM 3. *Let G be a π-solvable group with an S_π-subgroup K of G. Assume that $O_{\pi'}(G) = 1$ and that $A \in A_2(K) \cap A_\infty(K) \neq \varnothing$, then*