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ISOMORPHISMS BETWEEN HARMONIC AND P-HARMONIC HARDY SPACES ON RIEMANN SURFACES

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In this paper we investigate the relationship between the Hardy space $H^q(R)$ of harmonic functions on a hyperbolic Riemann surface R, and the Hardy space $P^q(R)$ of solutions of the equation $\Delta u = Pu$, where $P \geqq 0$, $P \not\equiv 0$ is a C^1-density on R. Under certain conditions these spaces are shown to be canonically isomorphic, although in general this is not the case. However, specific subspaces are found which are isomorphic and their relationship with other function spaces is discussed.

1. Introduction. Hardy spaces on Riemann surfaces have been studied by Heins [2], Schiff [11], in the setting of Φ-bounded functions by Parreau [8], and in the general context of harmonic spaces by Lumer-Naim [4], among others. The present work, in the setting of a hyperbolic Riemann surface R, examines the Hardy space $P^q(R)$ for the equation $\Delta u = Pu$, $P \geqq 0$, $P \not\equiv 0$, which thus falls within the framework of [4]. Hence, the results contained therein will be applicable to our study of the isomorphic relations between the harmonic Hardy space $H^q(R)$ and $P^q(R)$. In general, no such isomorphic relation between $H^q(R)$ and $P^q(R)$ exists, yet when particular subspaces are considered, an isomorphism is shown to indeed exist between the specified subspaces.

In the fourth section we investigate the conditions under which the inclusion relations are strict or not, between the various spaces which have been introduced. Certain isomorphisms are obtained under new conditions in the final section.

Some of the results obtained have natural generalizations to harmonic spaces and to Φ-bounded P-harmonic functions for a convex increasing Φ, however, these aspects of the theory will not be treated here.

2. Preliminaries. Let R be a hyperbolic Riemann surface and $P \geqq 0$, $P \not\equiv 0$ a C^1-density on R. We denote by $PB(R)$ (resp. $HB(R)$) the space of bounded C^2-solutions on R of the elliptic equation $\Delta u = Pu(\Delta u = 0)$, and by $PB'(R)$ (resp. $HB'(R)$) the quasibounded counterpart. A C^2-solution of $\Delta u = Pu$ is called a *P-harmonic* function, and the space of such functions on an open set U of R is denoted by $P(U)$. $H(U)$ denotes the space of harmonic functions on U. A