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ABELIAN GROUPS IN WHICH EVERY ENDOMORPHISM IS A LEFT MULTIPLICATION

W. J. WICKLESS

Let $\langle G+\rangle$ be an abelian group. With each multiplication on G (binary operation $*$ such that $\langle G+*\rangle$ is a ring) and each $g \in G$ is associated the endomorphism g_l^* of left multiplication by g. Let $L(G)=\{g_l^* \mid g \in G,\ * \varepsilon \text{ Mult } G\}$. Abelian groups G such that $L(G)=E(G)$ are studied. Such groups G are characterized if G is torsion, reduced algebraically compact, completely decomposable, or almost completely decomposable of rank two. A partial results is obtained for mixed groups.

Let $\langle G+\rangle$ be an abelian group. With each multiplication on G (binary operation $*$ such that $\langle G+*\rangle$ is a ring) and each $g \in G$ is associated the endomorphism g_l^* of left multiplication by g given by $g_l^*(x) = g * x$, $x \in G$. Let $L(G)$ be the set of all such endomorphisms, i.e., $L(G) = \{g_l^* \mid g \in G,\ * \varepsilon \text{ Mult}\,(G)\}$. In general all one can say is that $L(G)$ is a subset of the endomorphism ring $E(G)$. In this paper we consider abelian groups G such that every endomorphism is a left multiplication.

DEFINITION 1. An abelian group G is multiplicatively faithful iff $L(G) = E(G)$.

We mostly follow the notations in [2]. Specifically: all groups are abelian, rings are not necessarily associative, $\otimes$ denotes the tensor product over Z and $g\otimes\_$ the natural map $x \to g \otimes x$ from G into $G \otimes G$, $o(x)$ is the order of an element x, $Z(d)$ is the cyclic group of order d and $Z(d)^*$ is the multiplicative group of units in $Z(d)$. For a prime p, we write Z_p for the localization of Z at p and $\hat{Z}_p$ for the ring (or group) of p-adic integers. We use $t(A)[t(x)]$ for the type of a rank one torsion free group A [element x] and $h(x)$ for the height sequence. Finally, $\langle S\rangle[\langle S\rangle_*]$ is the subgroup [pure subgroup] generated by S.

We begin by listing some simple results.

A. Let θ_g: Hom $(G \otimes G, G) \to E(G)$ be given by $\theta_g(\Delta) = \Delta \circ (g \otimes\_)$, $\Delta \in \text{Hom}\,(G \otimes G, G)$, $g \in G$. Then G is multiplicatively faithful iff $\bigcup_{g \in G} \text{Image}\, \theta_g = E(G)$.

Proof. Mult G, the group of all multiplications on G, is isomorphic