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ARCHIMEDEAN AND BASIC ELEMENTS IN COMPLETELY DISTRIBUTIVE LATTICE-ORDERED GROUPS

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It is known that the *bi*-prime group $B(G)$ of an l-group G contains the basic elements of G. We show that every l-group G possesses a unique, maximal, archimedean, convex l-subgroup $A(G)$, and that if G is completely distributive and if $A(G)^{\perp}$ is representable, then $B(G)$ has a basis.

1. Introduction. An element s of a lattice-ordered group (l-group) G is *basic* (see [4]) if $s > 0$ and the closed interval $[0, s]$ is totally ordered. An l-group G *has a basis* if every $g > 0$ exceeds some basic element (any maximal disjoint set of basic elements is then a *basis*). An l-group G is *completely distributive* (see [3], [4], [9], [10]) if the relation

$$\bigwedge \{\vee \{g_{ij} \mid j \in J\} \mid i \in I\} = \bigvee \{\wedge \{g_{i(if)} \mid i \in I\} \mid f \in J^I\}$$

holds whenever $\{g_{ij} \mid i \in I, j \in J\} \subseteqq G$ is such that all the indicated joins and meets exist. By [5], p. 5.18, Theorem 5.8, every l-group which has a basis is completely distributive. For *archimedean* l-groups, i.e. those in which $a \geqq nb \geqq 0$ for all natural numbers n implies $b = 0$, more can be said: viz., an archimedean l-group has a basis if and only if it is completely distributive ([5], p, 5.21, Theorem 5.10). In [8], we constructed, via minimal prime subgroups, the *bi*-prime group $B(G)$ of an l-group G (see §3 below) which contains all the basic elements and which, if G is completely distributive and representable, has a basis. In this note, we introduce "archimedean elements" (see §2 below) in order to investigate possible connections among the above results. Thus, in §2, we show that every l-group G posseses a unique, maximal, archimedean, convex l-subgroup $A(G)$. (Kenny [7] independently proved this result for representable l-groups.) It follows that if $A(G)^{\perp} = \{0\}$, then G is completely distributive if and only if G has a basis. In §3, proving somewhat more general results, we show that $A(B(G)) = B(A(G))$ and hence that if G is completely distributive and if $A(G)^{\perp}$ is representable, then $B(G)$ has a basis. In §4, we construct two examples, one of which is of completely distributive, nonrepresentable l-group which has a basis and for which $A(G)^{\perp}$ is representable.

NOTATION AND TERMINOLOGY. We use $\square$ for the empty set and write functions on the right. We use N, Z, and R for the natural