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FUNCTIONAL RELATIONSHIPS BETWEEN A SUBNORMAL OPERATOR AND ITS MINIMAL NORMAL EXTENSION

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Let K be a compact subset of the plane. $C(K)$ denotes the continuous functions on K and $R(K)$ denotes those continuous functions of K which are uniform limits of rational functions whose poles lie off K. We say that f is minimal on K if $f \in R(K)$ and for every complex number c

$$R(L_c) = C(L_c)$$

where $L_c = \{z \in K \mid (fz) = c\}$.

Let S be a subnormal operator on a Hilbert space $\mathscr{H}$ with its minimal normal extension N on the Hilbert space $\mathscr{K}$. The spectrum of S is denoted by $\sigma(S)$. In this paper it is shown that if f is minimal on $\sigma(S)$ then $f(N)$ on $\mathscr{K}$ is the minimal normal extension of $f(N)$ restricted to $\mathscr{H}$. Some new results about subnormal operators follow as corollaries of this theorem.

An operator S acting on a Hilbert space $\mathscr{H}$ is called subnormal if there exists a normal operator N acting on a Hilbert space $\mathscr{K}$, which contains $\mathscr{H}$, such that $Nx = Sx$ for all x in $\mathscr{H}$. N is called the minimal normal extension (abbreviated mne.) of S when $\mathscr{K}$ is the only closed subspace containing $\mathscr{H}$ that reduces N. This is equivalent to saying that the closure of the linear manifold

$$\left\{\sum_{j=0}^{n} N^{*j} x_j \mid x_j \in \mathscr{H}, n \text{ a nonnegative integer}\right\}$$

is all of $\mathscr{K}$. (For the elementary properties of subnormal operators consult [2, 5].)

If K is a compact set in the plane then $C(K)$ denotes the continuous functions on K and $\mathscr{A}(K)$ is the collection of functions f analytic on some open set $G(f) \supset K$. $P(K)$ and $R(K)$ are the uniform closures of the polynomials and rational functions with poles off K, respectively. Further, ∂K and int K denote the boundary and interior of K, respectively. $\hat{K}$ designates the polynomial convex hull of K. [3, p. 66].

The set of bounded operators on a (complex) Hilbert space $\mathscr{H}$ is denoted by $\mathscr{B}(\mathscr{H})$ and $\sigma(T)$ represents the spectrum of any operator T belonging to $\mathscr{B}(\mathscr{H})$. Finally, $\boldsymbol{C}$ denotes the complex numbers and $\boldsymbol{N}$ denotes the nonnegative integers.

2. The problem. Throughout the rest of this paper it will be