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SPACES OF DISCRETE SUBSETS OF A LOCALLY COMPACT GROUP

N. OLER

This paper represents a continuing effort to develop elements of a general theory of packing and covering by translates of a fixed subset of a group. For P a subset of a group X a subset L is a left P-packing if for any distinct elements x_1 and x_2 in L, $x_1P \cap x_2P$ is empty. A subset M of X is a left P-covering if $MP = X$. The Chabauty topology on the set of discrete subgroups of a locally compact group has been used only to a rather limited extent in the Geometry of Numbers but by its very definition is a natural one to work with in studying packing and covering problems. The main results of this paper are that the Chabauty topology extends to the family of all closed discrete subsets containing the identity and that if X is σ-compact then $S(X, P)$ the space of all left P-packings for a fixed neighborhood P of the identity is locally compact.

The Chabauty topology [1] extends in the following way. Let X be a Hausdorff locally compact group with identity e and $\mathscr{N}$ the family of open neighborhoods of e. We introduce the following notation: For any three subsets A, B and C of X we define $L(A, B, C)$ to be the set of subsets, A', of X satisfying

$$A' \cap B \subset AC \text{ and } A \cap B \subset A'C$$

and $R(A, B, C)$ as the set of subsets A' of X satisfying

$$A' \cap B \subset CA \text{ and } A \cap B \subset CA' .$$

DEFINITION 1. We denote by $\mathscr{S}(X)$ the space of closed discrete subsets of X containing e with the topology generated by $\{L(H, K, U)$: $H \in \mathscr{S}(X)$; K a compact subset of X; $U \in \mathscr{N}\}$.

The following two assertions are immediate consequences of the definition of L.

PROPOSITION 1. *If* $U_1 \supset U_2$ *then* $L(H, K, U_1) \supset L(H, K, U_2)$.

PROPOSITION 2. *If* $K_1 \supset K_2$ *then* $L(H, K_1 U) \subset (H, K_2, U)$.

As a further consequence we have