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MULTIPLIERS ON A BANACH ALGEBRA WITH A BOUNDED APPROXIMATE IDENTITY

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Let A be a Banach algebra with a bounded approximate identity $\{e_\alpha \mid \alpha \in \Lambda\}$, and $M(A)$ the multiplier algebra on A. In this paper, we obtain a representation for $M(A)$ such that each multiplier operator appears as a multiplicative operator. The proof makes use of the weak-* compactness of the net $\{Te_\alpha \mid \alpha \in \Lambda\}$ and the algebraic properties of a multiplier.

1. Introduction. In 1951, J. G. Wendel showed that the left centralizers on $L_1(G)$, G a locally compact group, was equivalent to $C_0(G)^*$, the space of regular Borel measures on G. Thus, if T is a centralizer and x is any element in $L_1(G)$ then $Tx = \xi * x$ for some Borel measure ξ. It is also well known that if A is a Banach algebra with an identity element then any multiplier on A is determined by its action on the identity element. In this paper, we show that if A is a Banach algebra with a bounded approximate identity then there exist a continuous isomorphism of A such that each multiplier defined on A is given by point-wise multiplication. In the case that the approximate identity is uniformly bounded by one, the representation is norm preserving. Thus we obtain an isometric isomorphism for all multipliers on $L_1(G)$ and for all multipliers on any B^*-algebra such that the action of a multiplier is given by point-wise multiplication by a fixed element in A.

2. The representation space for $M(A)$.

DEFINITION 2.1. Let A be a Banach algebra and T a mapping from A into A. The map T is a multiplier provided

$$x(Ty) = (Tx)y \qquad (x, y \in A) .$$

Every multiplier turns out to be a continuous function and the set of all multipliers on A under pointwise operations is a commutative subalgebra of $B(A)$, the set of all bounded linear operators on A([5]).

NOTATION 2.2. In this paper, a Banach algebra with a bounded approximate identity will be denoted by A and the multiplier algebra on A will be denoted by $M(A)$. For any Banach algebra X, we denote the weak-* convergence of a net in X^*, the dual space of X, indexed by $\alpha \in \Lambda$, by "$\lim_\alpha^{wk-*}(\cdot)$". Unless otherwise stated, we denote the bound on the approximate identity by M.