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GEVREY CLASSES AND HYPOELLIPTIC BOUNDARY VALUE PROBLEMS

RALPH A. ARTINO

Let $P(D, D_t)$ be a hypoelliptic differential operator with constant coefficients of type μ with index of hypoellipticity equal to $d \geqq 1$. Let Ω be an open subset of the half space R_+^{n+1} with plane piece of boundary ω contained in R_0^n. Let $Q_1(D, D_t), \cdots, Q_\mu(D, D_t)$ be μ partial differential operators with constant coefficients and consider the boundary value problem:

$$(1)\qquad \begin{aligned} &P(D, D_t)u = f \quad \text{in } \Omega \\ &Q_\nu(D, D_t)u\,|_\omega = g_\nu \qquad 1 \leqq \nu \leqq \mu\,. \end{aligned}$$

In this paper necessary and sufficient conditions are given on $Q_1, \cdots, Q_\mu$ in order that all solutions u of (1) shall belong to the Gevrey class of index d in $\Omega \cup \omega$ whenever the initial data belong to such classes of functions. In particular, we give not only algebraic conditions but also show how to construct a parametrix for such problems.

Introduction. In 1958 Hörmander first studied regular boundary value problems, giving necessary and sufficient conditions for solutions of (1) to be C^∞. (see Hörmander [8]). There, Hörmander gives an algebraic characterization based on the variety of zeros of the characteristic function of the boundary value problem. Later on, it was shown that fundamental solutions to elliptic boundary value problems can be constructed with the aid of this characteristic function (see J. Barros-Neto [4, 5]). Moreover, it can be used to construct a parametrix for hypoelliptic problems (see J. Barros-Neto [6]). In this paper the characteristic function is used to give an algebraic characterization of d-hypoelliptic problems. In doing so a different technique is used than that in [8] in order to get more refined estimates. Consequently the special result obtained in [8] for elliptic operators is obtained here. The results here can be extended to Gevrey classes which distinguish the rate of growth of derivatives in different directions. (see [7]). Furthermore, these results have many applications to semi-elliptic problems (see [2]).

The plan of this paper is as follows: In § 1 d-hypoelliptic boundary value problems are defined and the main results are stated. In § 2 the first two equivalences are proved. In § 3 we make use of the parametrix of the boundary value problem and conclude the proof of the main results in § 4.

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