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# BOUNDS FOR NUMBERS OF GENERATORS OF COHEN-MACAULAY IDEALS

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**Let $(R, \underline{m})$ be a local Cohen-Macaulay ring of dimension $d$ and multiplicity $e(R) = e$. A natural question to ask about an $\underline{m}$-primary ideal $I$ is whether there is any relation between the number of generators of $I$ and the least power $t$ of $\underline{m}$ contained in $I$. ($t$ will be called the nilpotency degree of $R/I$) It is quite straight forward to obtain a bound for $v(I)$, the number of generators in a minimal basis of $I$, in terms of $t$ and $e$. However, there are several interesting applications. The first is the existence of a bound for the number of generators of any Cohen-Macaulay ideal $I$, i.e. any ideal $I$ such that $R/I$ is Cohen-Macaulay, in terms of $e(R/I)$, $e(R)$ and height $I$. The second application is a bound in terms of $d$ and $e$ for the reduction exponent of $\underline{m}$.**

1. $\underline{m}$-primary ideals. In this section we will use only the standard facts about the existence and properties of superficial elements. However, later we will need a result stronger than the usual existence theorem for these elements so we take this opportunity to recall the definition and prove this special form of the existence theorem.

DEFINITION. Let $(R, \underline{m})$ be a local ring. An element $x$ in $\underline{m}$ is superficial for $\underline{m}$ if there is an integer $c > 0$ such that

$$(\underline{m}^n : x) \cap \underline{m}^c = \underline{m}^{n-1} \quad \text{for all} \quad n > c\ .$$

It is a standard fact that $x$ is superficial for $\underline{m}$ if and only if there is an integer $c > 0$ such that $0 \neq \bar{X} \in \underline{m}/\underline{m}^2 = G_1$ and

$$(0 : \bar{x}G) \cap G_n = 0$$

for $n \geqq c$ where $G_n = m^n/m^{n+1}$, and $G = G_0 \oplus G_1 \oplus \cdots$.

LEMMA 1.1. *Let $(R, \underline{m})$ be a local ring with $R/\underline{m}$ infinite. Let $I, J_1, \cdots, J_s$ be distinct ideals of $R$ which are also distinct from $\underline{m}$. Then there is an element $x$ in $R$ such that*

(1) $x \notin J_i\ ,$ $\qquad i = 1, \cdots, s$

(2) *$x$ is a superficial element for $\underline{m}$*

*and*

(3) *the image of $x$ in a superficial element for $\underline{m}/I$.*

*Proof.* Let $G = R/\underline{m} \oplus \underline{m}/\underline{m}^2 \oplus \cdots$ and $\bar{G} = R/\underline{m} \oplus \underline{m}/\underline{m}^2 + I \oplus$