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MATRIX TRANSFORMATIONS AND ABSOLUTE SUMMABILITY

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The main results of this paper are two theorems which give necessary conditions for a matrix to map into ℓ the set of all subsequences (rearrangements) of a null sequence not in ℓ. These results provide affirmative answers to the following questions proposed by J. A. Fridy. Is a null sequence x necessarily in ℓ if there exists a sum-preserving $\ell - \ell$ matrix A that maps all subsequences (rearrangements) of x into ℓ?

1. Introduction. Let s, m, c, c_0 and cs denote, respectively, the set of all complex sequences, the set of all bounded sequences in s, the set of all convergent sequences in s, the set of all null sequences in c, and the set of all sequences in s with sequence of partial sums in c. Let

$$\ell = \{x \in s \colon \Sigma \,|\, x_p \,| < \infty\} \text{ and } \ell^2 = \{x \in s \colon \Sigma \,|\, x_p \,|^2 < \infty\} .$$

A matrix A which maps each element of ℓ into ℓ is called an $\ell - \ell$ matrix and may be characterized [3] and [6] by the property: $\{\sum_{p=1}^{\infty} | a_{pq} |\}_{q=1}^{\infty} \in m$. If, in addition, $\sum_{p=1}^{\infty} \sum_{q=1}^{\infty} a_{pq} x_q = \sum_{q=1}^{\infty} x_q$, whenever $x \in \ell$, then A is a sum-preserving $\ell - \ell$ matrix; this is characterized by $\sum_{p=1}^{\infty} a_{pq} = 1$, for each q.

In 1943, R. C. Buck [1] showed that a sequence x is convergent if some regular matrix sums every subsequence of x. J. A. Fridy [5] has obtained an analog to Buck's theorem in which "subsequence" is replaced by "rearrangement." In addition, he has characterized ℓ by showing that $x \in \ell$ if there is a sum-preserving $\ell - \ell$ matrix that transforms every rearrangement of x into ℓ. In § 2 of the present paper, necessary conditions are obtained for a matrix to map into ℓ the set of all subsequences of a null sequence not in ℓ. This result yields as a corollary the affirmative answer to the following question proposed by J. A. Fridy [5]. Is a null sequence x necessarily in ℓ if there exists a sum-preserving $\ell - \ell$ matrix that maps all subsequences of x into ℓ? In § 3, necessary conditions are obtained for a matrix to map into ℓ all rearrangements of a null sequence not in ℓ. This yields as a corollary Fridy's characterization of ℓ mentioned above. Finally, § 4 contains examples of matrix mappings involving both subsequences and rearrangements.

2. Subsequences. The following two lemmas will be instru-