PACIFIC JOURNAL OF MATHEMATICS
Vol 64, No 1, 1976

SPECTRAL APPROXIMATION THEOREMS IN LOCALLY CONVEX SPACES

VOLKER WROBEL

We present some results on collectively compact operator approximation theory in locally convex Hausdorff spaces (l.c.s.). The notion of a collectively compact family of operators acting on a Banach space has been introduced by Anselone and Palmer in connection with the numerical solution of integral equations. Meanwhile collectively compact families of operators have been studied in general topological vector spaces. In contrast to those investigations dedicated to the characterization of collectively compact families of operators the present paper focuses on spectral approximation theorems in l.c.s. similar to those given by Anselone and Palmer in the case of Banach spaces. In doing this it turns out that the notion of the spectrum, which causes no problems in Banach algebra theory, entails some difficulty. A way out is indicated by using notions and tools of locally convex algebra theory.

0. Notations. Throughout this paper let E denote a l.c.s. over the field of complex numbers $\mathbf{C}$. E is always assumed to be equipped with a basis $\mathcal{P}$ of continuous seminorms p. By $\mathcal{U}_p$ we denote the closed, convex, and circled neighborhood of zero $\{x \in E: p(x) \leqq 1\}$ in E. Let $\mathcal{L}_s(E)$, $\mathcal{L}_c(E)$ and $\mathcal{L}_b(E)$ denote the locally convex algebra of all continuous linear operators on E equipped with the topology of uniform convergence on finite, compact, and bounded subsets of E.

The formulation of spectral approximation theorems requires some remarks on the notion of spectrum. In contrast to Banach algebra theory there are different ways for introducing a spectrum for the elements of a locally convex algebra, which in general lead to different sets. For $T \in \mathcal{L}_b(E)$ a straightforward generalization from the theory of Banach algebras would lead to the following notions: Denote by $\rho_B(T) := \{z \in \mathbf{C}: (z \cdot \mathrm{id}_E - T)^{-1} \in \mathcal{L}(E)\}$ resp. $\sigma_B(T) := \mathbf{C} \backslash \rho_B(T)$ the *Banach-resolvent set* resp. the *Banach-spectrum* of T. These notions which are of great importance for solving eigenvalue problems for the linear operator T unfortunately are not suitable for involving such a powerful tool as the analytic functional calculus for general l.c.s. E. That is why we introduce the following notions current in locally convex algebra theory (see [1]). For $T \in \mathcal{L}_b(E)$, the *spectrum* $\sigma(T)$ of T is the complement in the Riemann sphere $\hat{\mathbf{C}}$ of the largest open set $\rho(T)$ in which $z \mapsto R(z, T) := (z \cdot \mathrm{id}_E - T)^{-1}$ is locally holomorphic (in the