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SEQUENCINGS AND STARTERS

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B. Gordon characterized sequenceable Abelian groups as those Abelian groups with a unique element of order 2. In this paper Gordon's argument is generalized to prove that there are non-Abelian sequenceable groups of arbitrarily large even order. It is also noted that the sequencings described by Gordon are related to 1-factorizations of complete graphs and to Howell Designs.

1. Introduction.

Suppose G is a finite group of order n with identity e. A *sequencing* of G is an ordering $e, a_2, \cdots, a_n$ of all the elements of G such that the partial products $e, ea_2, ea_2a_3, \cdots, ea_2 \cdots a_n$ are distinct and hence also all of G. Sequencings arose in connection with the problem of constructing complete Latin Squares [5]. Later [8] it was noticed that sequencings can be used to decompose complete directed graphs into directed Hamiltonian paths. Other possible uses of sequencings are described here. It turns out that the sequencings of Gordon all induce 1-factorizations of an appropriate complete graph via associated "starters" [6, p. 176–177]. Certain sequencings and their "starters" also induce Howell Designs of type $H(2m-2, 2m)$ by the "starter-adder" method [6, 176–177]. Thus, it appears that sequencings might have a broader applicability that has yet been recognized.

As mentioned above, sequenceable Abelian groups have been characterized [5]. But the sequencing question for non-Abelian groups has hardly been budged. Keedwell [7] reports that there are 9 known sequenceable non-Abelian groups and apparently in 7 of these cases, the results were determined by computer. Recently [2] other non-Abelian groups have been shown sequenceable. In this paper known sequencings and Gordon's original argument are used to construct infinite families of sequenceable non-Abelian groups of even order.

The sequencings we construct have the following property.

DEFINITION 1. Suppose G is a group of order $2n$ with identity e and unique element g^* of order 2. A sequencing $e, a_2, \cdots, a_n, \cdots, a_{2n}$ will be called a *symmetric sequencing* iff $a_{n+1} = g^*$ and for $1 \leqq i \leqq n-1$, $a_{n+1+i} = (a_{n+1-i})^{-1}$.

If g^* is the unique element of order 2 in G, then g^* is in the center of G. Thus, symmetric sequencings

$$S\colon e, a_2, \cdots, a_n, a_{n+1}, a_n^{-1}, \cdots, a_2^{-1}$$