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FINITELY GENERATED PROJECTIVE MODULES AND TTF CLASSES

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Let P be a finitely generated projective right A-module with trace ideal T and A-endomorphism ring B. Associated with P are the TTF classes, $\mathscr{T}_F = \{{}_AX \mid P \otimes X = 0\}$ and $\mathscr{T}_H = \{X_A \mid \mathrm{Hom}(P, X) = 0\}$. An investigation of these TTF classes yields characterizations of various conditions on P and T; e.g., (1) ${}_BP$ is projective (flat) and (2) ${}_AT$ is projective (flat). The concept of weak stability for a hereditary torsion class is introduced and characterizations are given.

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1. Preliminaries.

In this paper all rings will be associative with unit and all modules will be unitary. $E(M)$ will denote the injective hull of a module M. Given a ring A the category of all left (right) A-modules will be denoted by ${}_A\mathscr{M}(\mathscr{M}_A)$.

A familiarity with torsion theories and their terminology is assumed. For further information the reader is referred to **[5]** or **[14]**. Given a hereditary torsion class $\mathscr{T}$, its associated idempotent topologizing filter will be denoted by $f(\mathscr{T})$. We let $t(X)$ denote the torsion submodule of a module X.

Jans **[7]** has called a torsion class $\mathscr{T}$ which is also a torsionfree class for some torsion class $\mathscr{C}$, a torsion-torsionfree (TTF) class. In this case we have a TTF-theory $(\mathscr{C}, \mathscr{T}, \mathscr{F})$. In **[7]** it is shown there is a one-to-one correspondence between the TTF classes of ${}_A\mathscr{M}$ and the idempotent ideals of A given by $\mathscr{T} \to T = c(A)$, the $\mathscr{C}$-torsion submodule of A. The inverse correspondence is given by $T \to \mathscr{T} = \{{}_AX \mid TX = 0\}$. One easily checks that $\mathscr{C} = \{{}_AX \mid A/T \otimes X = 0\}$, $\mathscr{F} = \{{}_AX \mid \mathrm{Hom}(A/T, X) = 0\}$, and T is the smallest element in $f(\mathscr{T})$ (i.e., $T \in f(\mathscr{T})$ and $T \subseteq I$ for all $I \in f(\mathscr{T})$).

For an A-module U, we say that an A-module X is of U-dominant codimension $\geqq n$ (written U.dom.codim.$X \geqq n$) if there is an exact sequence

$$X_n \to \cdots \to X_1 \to X \to 0$$

where each X_i is a direct sum of copies of U. This definition is dual to