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ON CONTINUOUS IMAGE AVERAGING OF PROBABILITY MEASURES

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Let M be a compact space, and X a complete sparable metric space. Let $P(X)$ denote the probability measures on X. Let λ be a probability measure on M. Define a function φ_λ from $C(M, P(X))$ to $P(X)$ by $\varphi_\lambda(T)(f) = \int T(t)(f)d\lambda(t)$ for every $T \in C(M, P(X))$, $f \in C(X)$. We show that φ_λ is an open mapping.

1. Introduction. By a measure on a space X, we mean a regular Borel measure on X. A nonnegative measure is called a probability measure if its total mass is 1.

Let M be a compact space, and let X be a complete separable metric space. Let $P(X)$ denote the collection of all probability measures on X. Let $C(X)$ denote the set of all bounded continuous real-valued functions on X. Give $P(X)$ the weak topology as functionals on $C(X)$. Let $C(M, P(X))$ denote the set of all continuous functions from M into $P(X)$. Give $C(M, P(X))$ the topology of uniform convergence. Let λ be a fixed probability measure on M. For each $T \in C(M, P(X))$, define a functional $\varphi_\lambda(T)$ on $C(X)$ by

$$\varphi_\lambda(T)(f) = \int T(t)(f)d\lambda(t) .$$

By [3, p. 35 and p. 47], $\varphi_\lambda(T)$ may be considered as a measure in $P(X)$. Write $\varphi_\lambda(T) = \int T(t)d\lambda(t)$. Denote the mapping $T \rightarrow \varphi_\lambda(T)$ by φ_λ. Then φ_λ is a continuous function from $C(M, P(X))$ into $P(X)$. This paper is to show that φ_λ is an open mapping. This result contains a result due to Eifler [2, Theorem 2.4] as a special case when M consists of two points.

For a metric space X, we write $x_n \rightarrow x$ if $(x_n)_{n=1}^{\infty}$ converges to x in X.

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2. Basic lemmas. We will use the following notation in Lemma 2.1: Let X and Y be complete separable metric spaces, and $\pi: Y \rightarrow X$ a continuous function. Then π induces a mapping also denoted by π, from $P(Y)$ to $P(X)$ and defined by $\pi\mu(E) = \mu(\pi^{-1}(E))$.