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PACKING SPHERES IN ORLICZ SPACES

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A collection of open balls of radius r can be packed in the unit ball U of a Banach space provided each ball is a subset of U and the intersection of any two is empty. In an infinite dimensional Banach space, it is possible to find a largest number Λ so that if $r \leqq \Lambda$ then an infinite number of spheres of radius r can be packed in U. In this paper, upper and lower bounds are found for this number in Orlicz spaces.

For the space l_2, this number was found by Rankin [7] to be $1/(1+\sqrt{2})$ and this result was extended in [1] to show that the number in $l_p (1 \leqq p < \infty)$ is $1/(1+2^{1-1/p})$. In 1970 Kottman [4] showed that $1/3 \leqq \Lambda \leqq 1/2$ for any Banach space. More recently, Wells and Williams [10] used a generalized Riesz-Thorin interpolation theorem to obtain the exact value of Λ in the $L^p(\mu)$ $(1 \leqq p < \infty)$ spaces with some restrictions on the measure space when $2 < p < \infty$. The results in this paper include all the above and also show that all restrictions can be removed in the L^p case. Recent results have demonstrated that the structure of Orlicz spaces is quite different from L^p spaces and very little seems to be known in the Orlicz case. The packing criteria lead to some results on isometric embeddings of subspaces and to notions of noncompactness.

2. **Preliminaries.** An Orlicz function M will be a continuous convex nondecreasing function defined for $x \geqq 0$ and such that $M(0)=0$, $M(\infty)=\infty$ and $M(x)>0$ for $x>0$. The Orlicz space $L_M(X, \mathscr{A}, \mu)(=L_M)$ is the set of measurable scalar-valued functions defined on the measure space $(X, \mathscr{A}, \mu)$ such that $f \varepsilon L_M$ if and only if $\|f\|' < \infty$ where

$$\|f\|'_M = \inf \left\{k > 0 \colon \int_X M\left(\frac{|f|}{k}\right) d\mu \leqq 1\right\} .$$

For each Orlicz function M, a complementary function N is defined by

$$N(x) = \sup \{xy\text{-}M(y) \colon 0 < y < \infty\} .$$

If $M(x) = \int_0^x p(t)dt$ where p is a right continuous nondecreasing function, then $N(p(x)) = xp(x) - M(x)$ (cf [5]). Using this function, another norm can be defined on L_M