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SOME n-ARC THEOREMS

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G. T. Whyburn gave an inductive proof of the n-arc theorem for complete, locally connected, metric spaces. In this note Whyburn's proof is modified to generalize this theorem to the class of regular, T_1, locally connected spaces. This result is then used to obtain an affirmative solution to a conjecture of J. H. V. Hunt.

Our notation will follow that of Whyburn **[3]** and Hunt **[1]**.

Let X be a topological space and let P and Q be disjoint closed sets in X. A set C is said to *separate P and Q in the broad sense in X* if $X\setminus C = A \cup B$ where A is separated from B, $P\setminus C \subset A$ and $Q\setminus C \subset B$. The space X is said to be *n-point strongly connected between P and Q* if no subset of X with fewer than n points separates P and Q in the broad sense in X. A subset of X is said to *join P and Q* if some component of the set meets both P and Q.

1. The second n-arc theorem.

THEOREM 1. *The locally connected, regular, T_1 space X is n-point strongly connected between two disjoint closed sets P and Q if and only if there exist n disjoint open sets in X which join P and Q.*

Proof. The sufficiency is obvious. We shall prove necessity by induction on n. The case $n = 1$ follows from the fact that the components of X are open (as X is locally connected) and hence some component of X meets both P and Q. Suppose the theorem holds for all positive integers less than n.

Suppose X is n-point strongly connected between the disjoint closed sets P and Q. Let S denote the set of all $x \in X$ such that there exists a set S_x which is the union of n disjoint open connected sets $n-1$ of which join P and Q and the nth one joins P and x. Then S is clearly open in X. If $y \in X$ then $X\setminus\{y\}$ is $(n-1)$-point strongly connected between $P\setminus\{y\}$ and $Q\setminus\{y\}$. By induction $X\setminus\{y\}$ contains a set $U_1, \cdots, U_{n-1}$ of disjoint open connected sets joining P and Q. Since X is regular and locally connected there exist by the chaining lemma open connected sets $V_1, \cdots, V_{n-1}$ such that for each i $\bar{V}_i \subset U_i$ and V_i joins P and Q. The sets $V_1, \cdots, V_{n-1}$ have closures which are disjoint from y and from each other. If $y \in P$ then y is clearly in S so $P \subset S$.