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GENERALIZED MONOFORM AND QUASI-INJECTIVE MODULES

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For a torsion radical F (arising from an idempotent filter of right ideals) and a unital right R-module M over a ring R, let DM be the F-divisible hull $DM/M = F\,(EM/M)$, where EM is the injective hull of M.

Let $0 \neq \beta\colon N \to M$ be any nonzero homomorphism whatever from any F-dense submodule $N \subseteq M$. Then M is F-quasi-injective if each such β extends to a homomorphism of $M \to M$; M is F-monic if β is monic; M is F-co-monic if $\beta N \subseteq M$ is F-dense.

Each module M has a natural F-quasi-injective envelope $\mathbf{J}M$ inside $M \subseteq \mathbf{J}M \subseteq DM$.

THEOREM III. *Form the R-endomorphism rings $\Delta = \mathbf{End}\,\mathbf{J}M$ and $\Lambda = \mathbf{End}\,DM$, and $\Lambda^{\#} = \{\lambda \in \Lambda \mid \lambda M = 0\} \subseteq \Lambda$, the annihilator subring of M.*

When M is F-monic and F-co-monic and $FM = 0$, then

(1) $\Lambda^{\#}$ is exactly the annihilator $\Lambda^{\#} = \{\lambda \in \Lambda \mid \lambda\,\mathbf{J}M = 0\}$ of the submodule $\mathbf{J}M \subseteq DM$ and $\Lambda^{\#} \subseteq \Lambda$ is an ideal;

(2) $\Delta \cong \Lambda/\Lambda^{\#}$;

(3) Δ is a division ring.

For a torsion radical F and a torsion preradical G, let $\mathbf{I}M$ be the (F, G)-injective hull of M; and, more generally, Λ the ring of all those R-endomorphisms of $\mathbf{I}M$ with G-dense kernels. The above is derived as a special case where $G = 1$ is the identity functor and $\mathbf{I}M = DM$.

THEOREM II. **(i)** *M is (F, G)-quasi-injective $\Leftrightarrow \Lambda M \subseteq M$.*

(ii) *The (F, G)-quasi-injective hull $\mathbf{J}M$ of M exists and $\mathbf{J}M = M + \Lambda M$.*

(iii) *$\mathbf{J}M$ is the unique smallest (F, G)-quasi-injective module with $M \subseteq \mathbf{J}M \subseteq \mathbf{I}M$.*

Simple modules over a ring were at first generalized to quasi-simple modules (**[9]**), and then these to strongly uniform or monoform ones (**[15]** and **[6]**).

Here the monoform modules are further generalized to the F-monic ones. The quasi-injective hull plays an important role in the theory of quasi-simple modules (**[8]**, **[9]**, **[15]**, and **[6]**). Furthermore, there is a