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# ON SEMIPRIME P.I.-ALGEBRAS OVER COMMUTATIVE REGULAR RINGS

EFRAIM P. ARMENDARIZ

**Let $R$ be a commutative (von Neumann) regular ring with unit. This paper deals with algebras $A$ over $R$, and following standard conventions $A$ will be called a finitely generated $R$-algebra whenever $A$ is a finitely generated $R$-module. One of the principal results obtained is that all semiprime finitely generated $R$-algebras are regular rings. Combining this with a result of J. Wehlen and a theorem of G. Michler and O. Villamayor shows that the finitely generated semiprime algebras over commutative regular rings are precisely the semiprime central separable algebras over regular rings.**

Since any finitely generated algebra over a commutative ring satisfies a polynomial identity, (is a P.I.-algebra), this leads to consideration of semiprime P.I.-algebras with regular center. In general, these will only be semisimple $I$-rings. However if the center is also a self-injective ring then the algebra is $\pi$-regular; this fact is a consequence of the observation that every semiprime P.I.-algebra is weakly algebraic over its center in the sense that every element is a root of a nonzero polynomial with central coefficients.

It will be assumed throughout that the polynomial identities occurring have at least one coefficient $\pm 1$.

THEOREM 1. *Let $A$ be a semiprime finitely generated algebra over a commutative regular ring $R$. Then $A$ is a regular ring.*

*Proof.* For the proof we appeal to a theorem of J. Fisher and R. Snider [5, Theorem 1.1] which says that $A$ will be a regular ring provided

(i) $A/P$ is a regular ring for each prime ideal $P$ of $A$,

(ii) the union of any chain of semiprime ideals of $A$ is a semiprime ideal of $A$.

For (i), we first observe that any homomorphic image of $R$ is a regular ring and hence we may assume that $A$ is a faithful $R$-algebra with $R$ lying in the center of $A$. Now if $P$ is any prime ideal of $A$ then $P \cap R$ is a prime ideal of $R$ hence is a maximal ideal of $R$. Consequently, $A/P$ is a prime finite-dimensional algebra over the field $R/P \cap R$ and so $A/P$ is a simple Artinian ring. Thus (i) is satisfied. For (ii) suppose $A = Ra_1 + \cdots + Ra_t$, let $\{B_\lambda \mid \lambda \in \Lambda\}$ be a chain of semiprime ideals of $A$ and let $B = \bigcup_{\lambda \in \Lambda} B_\lambda$. Let $x \in A$ with $xAx \subseteq B$. Then there is some