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ON CAMERON AND STORVICK'S OPERATOR VALUED FUNCTION SPACE INTEGRAL

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In this paper "the probability density of path space" is introduced by the formula

$$p_\lambda^\alpha(t,u)=\frac{1}{2\pi}\int_{-\infty}^{\infty}\exp\left(-iu\eta-\frac{t}{\lambda}|\eta|^\alpha\right)d\eta,\qquad(\alpha>0).$$

If $\alpha=2$ and $\lambda>0$, $p_\lambda^\alpha(t,u)$ is the normal probability density. But if $\alpha>2$ this density can not be considered as a probability density. By this generalization, one can generalize the operator valued function space integral based on the Wiener integral.

Introduction. Cameron and Storvick introduced the operator valued function space integral in **[1]**. Johnson and Skoug **[9]** developed Cameron and Storvick's theory and improved the results obtained in **[1]**. To make the arguments in the following sections comprehensible, we will quote the operators $I_\lambda^\sigma(F)$ and $I_\lambda^{\mathrm{sec}}(F)$ from **[1]**, which played an important role in **[1]**, **[9]**. Let $B[a,b]$ denote the space of real valued functions on an interval $[a,b]$ which are continuous except for a finite number of finite jump discontinuities. Let $F(x)$ be a functional on $B[a,b]$ and $\psi\in L_2(-\infty,\infty)$, $\xi\in(-\infty,\infty)$. Then for $\operatorname{Re}\lambda>0$ and any partition σ: $a=t_0<t_1<\cdots<t_n=b$, the operator $I_\lambda^\sigma(F)$ is defined by the formula

$$\begin{aligned}(I_\lambda^\sigma(F)\psi)(\xi)=\lambda^{n/2}[(2\pi)^n(t_1-a)\cdots(t_n-t_{n-1})]^{-1/2}\int_{-\infty}^{\infty}(n)\int_{-\infty}^{\infty}\psi(v_n)\\ \cdot f_\sigma(\xi,v_1,\cdots,v_n)\exp\left(-\sum_{j=1}^{n}\frac{\lambda(v_j-v_{j-1})^2}{2(t_j-t_{j-1})}\right)dv_1\cdots dv_n\end{aligned}\tag{0.1}$$

where $v_0=\xi$, $f_\sigma(\xi,v_1,\cdots,v_n)=F[z(\sigma,\xi,v_1,\cdots,v_n,\cdot)]$,

$$z(\sigma,\xi,v_1,\cdots,v_n,t)=\begin{cases}v_j & \text{if}\quad t_j\leqq t<t_{j+1},\qquad j=0,1,\cdots,n-1,\\ v_n & \text{if}\quad t=b,\end{cases}$$

and where if n is odd we always choose $\lambda^{n/2}$ with nonnegative real part. Here $\int(n)\int$ means the n-fold integral. If $\lambda>0$, by using the Wiener integral, this can be written as