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SPACES WITH BASES SATISFYING CERTAIN ORDER AND INTERSECTION PROPERTIES

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An ortho-base is a base $\mathscr{B}$ such that the intersection of any subcollection is either an open set or a singleton for which the subcollection is a local base. This paper is primarily devoted to the relationship of bases of this sort to other topological properties of bases, and of the space itself. In §1, we compare ortho-bases to bases having similar properties, and study the relationship with developability and paracompactness. In §2, we define "rank" of a base for arbitrary cardinals and show how it is related to orthocompactness, ortho-bases, and bases of countable order. Section 3 treats two related ascending chain conditions in relation to bases of sub-infinite rank and ortho-bases. Section 4 relates the possession of an ortho-base to a number of "generalized metric" properties such as first countability, quasi-developability, and quasi-metrizability. The remaining two sections give examples illustrating the various properties and raise a number of unsolved problems.

1. Ortho-bases and related concepts.

Throughout this paper "space" will always mean "T_1 topological space." Many of the spaces we will be studying have bases satisfying a rather strong property:

DEFINITION 1.1. A base $\mathscr{B}$ for a space X is an *ortho-base* if for each subcollection $\mathscr{A}$ of $\mathscr{B}$, either (i) $\bigcap \mathscr{A}$ is open or (ii) $\bigcap \mathscr{A}$ is a nonisolated singleton $\{x\}$ and $\mathscr{A}$ is a base for the neighborhoods of x.

It is easy to see that every point in a space with an ortho-base has a totally ordered open base for its neighborhoods. The proof of the following lemma is also easy and is omitted.

LEMMA 1.2. *Let $\mathscr{B}$ be an ortho-base for a space X.*

(i) *Every subset of $\mathscr{B}$ which is a base is an ortho-base.*

(ii) *The collection of all unions of chains in $\mathscr{B}$ is an ortho-base.*

(iii) *The collection of all open intersections of subsets of $\mathscr{B}$ is an ortho-base.*

(iv) *Given a subspace Y of X, the collection of all sets of the form $Y \cap B$, with $B \in \mathscr{B}$ is an ortho-base for Y.*

Concepts similar to that of an ortho-base have appeared in the recent literature. For example, Alexandroff introduced the concept of a