

THE SPECTRA OF ENDOMORPHISMS OF ALGEBRAS OF ANALYTIC FUNCTIONS

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Suppose $0 < R < 1$, G is the open annulus $\{z \mid R < |z| < 1\}$ and $A(G)$ denotes the uniform algebra of functions analytic on G and continuous on \bar{G} . Each nonzero endomorphism T of $A(G)$ has the form $Tf = f \circ \varphi$ for some $\varphi \in A(G)$ with $\varphi(G) \subset G$. In the main result of this note, the spectra of endomorphisms of $A(G)$ are determined for the case where the inducing maps φ have a fixed point in G . In addition, further results are discussed for other algebras of analytic functions.

Introduction. In [2] we determined the spectra of a class of endomorphisms of the disc algebra $A(D)$, the uniform algebra of functions analytic on the open unit disc D and continuous on \bar{D} . In this note, other algebras of analytic functions are considered and the techniques and results of [2] are used to prove a generalization of the following theorem.

THEOREM A. *If T is a nonzero endomorphism of $A(D)$, then T has the form $Tf = f \circ \varphi$, for some $\varphi \in A(D)$ with $\varphi: \bar{D} \rightarrow \bar{D}$. If, moreover, the inducing function φ has a fixed point z_0 in D , then exactly one of the following three possibilities holds.*

(1) *φ is a schlicht map of D onto itself and T is an automorphism of $A(D)$. In this case $\sigma(T)$, the spectrum of T , is either the entire unit circle or else $\sigma(T)$ is a finite union of finite subgroups of the circle, or*

(2) *$\sigma(T) = \{\lambda \mid |\lambda| \leq 1\}$, or*

(3) *T^N is a compact operator for some positive integer N in which case $\sigma(T) = \{(\varphi'(z_0))^n \mid n \text{ is a positive integer}\} \cup \{0, 1\}$.*

The plan is to first prove that an analytic function φ which maps a bounded (open) region into itself has at most one fixed point unless φ is schlicht and onto. Knowing this, we consider an annular region G and the uniform algebra $A(G)$ of analytic functions on G which are continuous on \bar{G} and prove a theorem for $A(G)$ similar to Theorem A. Finally, we will indicate other regions for which similar results are valid and also state some later results concerning endomorphisms of the disc algebra.

1. Maps with two fixed points. In this section we prove