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UNBOUNDED COMPLETELY POSITIVE LINEAR MAPS ON C^*-ALGEBRAS

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We define unbounded, completely positive, operator valued linear maps on C^*-algebras, and investigate their natural order structure. Following F. Combes, J. Math. Pure et Appl., we study the quasi equivalence, equivalence and type of the Stinespring representations associated with unbounded completely positive maps. Following A. van Daele, Pacific J. Math., we study an unbounded completely positive map α with dense domain which is invariant under a group G of *-automorphisms and construct a G-invariant projection map ϕ' of the set $\mathscr{F}$ of continuous completely positive maps dominated by α, onto the set $\mathscr{F}_0$ of G-invariant elements of $\mathscr{F}_0$. This is used to derive various properties of the upper envelope of $\mathscr{F}_0$.

1. Introduction. We investigate the structure of unbounded completely positive linear maps on a C^*-algebra A. Our work generalises that of Combes **[3]** and van Daele **[15]** from scalar valued weights to operator valued ones. Haagerup **[9]** has also introduced a notion of an operator valued weight, which can be described as an unbounded conditional expectation. We note that an operator valued weight in the sense of Haagerup is automatically completely positive by an extension of [**9**, Lemma 4.5].

Recently, various authors **[10, 11, 12, 13]** with different applications in mind, have considered Stinespring-like constructions for certain positive definite operator-valued functions on involutive algebras. Rieffel **[13]** generalised the notion of a conditional expectation on C^*-algebras, and used their Stinespring representations to formulate a theory of induced representations of C^*-algebras. In **[11, 12]** Powers defined an unbounded *-representation of an involutive algebra, and obtained a Stinespring-decomposition with an unbounded *-representation for a completely positive linear map on a *-algebra with identity. Paschke **[10]** has also studied completely positive maps on *-algebras, and obtained the Stinespring decomposition for such a map on a unital *-algebra which is linearly spanned by its unitaries.

In §2 we construct the Stinespring representation for an unbounded completely positive linear map α on A, and begin an analysis of the natural order structure for such maps. In particular we study the family $\mathscr{F}$ of bounded, completely positive, linear maps majorised by α. In §3, when α has dense domain, we are concerned with the construction of a