

ABSOLUTELY DIVERGENT SERIES AND ISOMORPHISM OF SUBSPACES II

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The following relations between a Banach space E and a Banach space X are, roughly speaking, generalizations of the relation " E is a closed subspace of X ."

(LIX) The finite dimensional subspaces of E are uniformly isomorphic to subspaces of X under isomorphisms which extend to all of E without increase of norm.

(SpX) Finite rank mappings from any Banach space into E can be uniformly factored through subspaces of X .

(ASX) The continuous linear mappings from E into X distinguish the absolutely summing mappings from any Banach space into E .

(SIX) For each absolutely divergent series $\sum_n x_n$ in E there is a continuous linear mapping T from E into X such that $\sum_n Tx_n$ diverges absolutely.

Our main result is that these four conditions are equivalent if X contains a subspace isomorphic to $\lambda[X]$ where λ is a normal BK -space. A related result of some interest is that the class of continuous linear mappings which factor through spaces which contain a complemented copy of $\lambda[X]$ forms a Banach operator ideal.

The consideration of the above relations continues the theme begun in [2] and [7]. A similar result to our main result is proven in [7] under a different assumption on the space X — an isometric assumption. We do not know whether the hypothesis on X in the present paper is strictly weaker than that in the previous paper. But in this case it is an isomorphic assumption and easier to verify. For example, it is satisfied by any space with a symmetric basis.

1. Some prerequisites. A. *Sequence spaces.* The space of all sequences of scalars (s_i) (real or complex) with the product topology is denoted by ω . The subspace of ω which contains all sequences which are eventually 0 is denoted by φ . A Banach space λ of sequences is called a BK -space if the inclusion from λ into ω is continuous. A space of sequences λ is called normal if whenever (s_i) is in λ and (t_i) is in m , the BK -space of bounded sequences it follows that (ts_i) is also in λ . It is known that if λ is a BK space there is an equivalent norm $\|\cdot\|$ on λ for which