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SOME CELLULAR SUBSETS OF THE SPHERES

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Let H be a PL-homology sphere such that its multisuspension $S^k * H$ is topologically homeomorphic to the sphere. We prove that every cell-like subset of S^k is cellular. Also, every non-compact PL-manifold of dimension greater than four accepts uncountably many simplicial triangulations each of which contains a non-cellular k-simplex for every $k \neq 0$.

Introduction. The double suspension problem introduces many different simplicial triangulations for a PL-manifold; in particular, the case of the sphere. It is easy to see that these strange triangulations are not locally flat, however, we ask whether their simplexes are cellular. A positive answer is given for every cell-like subset of S^k, where S^k is the suspension sphere in $S^k * H$ (Theorem 1), and for every cell-like subset of a codimension-2 simplex which *properly* meets every nontrivial face of this simplex (Theorem 2). But it is negative for general cases (Theorem 4).

Finally, combining Theorem 4 and Theorem 5, it follows that there are uncountably many noncellular simplicial triangulations for a noncompact PL-manifold of dimension greater than four.

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DEFINITIONS AND NOTATION. Let $\mathbf{R}^n$ denote the n-dimensional Euclidean space, S^n the boundary $\partial\Delta^{n+1}$ of the standard $(n+1)$-simplex Δ^{n+1}.

Let X be a compact subset of a metric space M. X satisfies the *small loops condition* (SLC) if $UV(X \subset M)$ and there is a $\delta > 0$ such that each δ-loop in $V - X$ is null-homotopic in $U - X$ (see **[3]**).

For the notion of cellularity refer to Rushing **[8]** or McMillan **[7]**. A finite-dimensional compactum X is *cell-like* if X has the shape of a point (see **[6]**).

A *simplicial triangulation* of a space M is a pair (T, φ) where T is a simplicial complex and φ is a homeomorphism from $|T|$, the underlying space of T, onto M. Two triangulations (T, φ) and (T', φ') are said to be *equivalent* if there is a PL-homeomorphism $\psi: |T| \to |T'|$ such that $\varphi = \varphi'\psi$. (We refer to Hudson **[5]** for general notions in the PL-category.)