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A FORMULA FOR THE NORMAL PART OF THE LAPLACE-BELTRAMI OPERATOR ON THE FOLIATED MANIFOLD

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In this paper, we give a formula for the normal part of the Laplace-Beltrami operator with respect to the second connection on a foliated manifold with a bundle-like metric. This formula is analogous to the formula obtained by S. Helgason.

1. Itroduction. We shall be in C^∞-category and manifolds are supposed to be paracompact, connected Hausdorff spaces.

Let M be a complete $(p+q)$-dimensional Riemannian manifold and H a compact subgroup of the Lie group of all isometries of M. We suppose that all orbits of H have the same dimension p. Then H defines a p-dimensional foliation F whose leaves are orbits of H, and the Riemannian metric is a bundle-like metric with respect to the foliation F. A quotient space $B = M/F$ is a Riemannian V-manifold [5]. Let L_D be the Laplace-Beltrami operator on M with respect to the second connection D[8], and let $\Delta(L_D)$ denote the operator defined by (*) in § 4. Our goal in this paper is the following theorem:

THEOREM. *Let L_D be the Laplace-Beltrami operator on M with respect to the second connection D and L_B the Laplace-Beltrami operator on B with respect to the Levi-Civita connection associated with the Riemannian metric defined by the normal component of the metric on M. Then*

$$\Delta(L_D) = \delta^{-1/2} L_B \circ \delta^{1/2} - \delta^{-1/2} L_B(\delta^{1/2})$$

where δ is the function given by (**) *below.*

This theorem is analogous to the following result obtained by S. Helgason [2]: *Suppose V is a Riemannian manifold, H a closed unimodular subgroup of the Lie group of all isometries of V* (with the compact open topology). *Let $W \subset V$ be a submanifold satisfying the condition: For each $w \in W$,*

$$(H \cdot w) \cap W = \{w\}\ , \quad V_w = (H \cdot w)_w \oplus W_w\ ,$$

where $\oplus$ denotes orthogonal direct sum. Let L_V and L_W denote the Laplace-Beltrami operators on V and W, respectively. Then

$$\Delta(L_V) = \delta^{-1/2} L_W \circ \delta^{1/2} - \delta^{-1/2} L_W(\delta^{1/2})$$