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A CLASS OF MAXIMAL TOPOLOGIES

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In this note, we characterize maximal topologies of a class of topological properties which include lightly compact spaces and QHC-spaces and, when restricted to completely regular spaces, pseudocompact spaces. In addition we prove some results relating maximal lightly compact and maximal pseudocompact spaces.

A. B. Raha **[12]** has shown that maximal lightly compact spaces are submaximal as are maximal pseudocompact spaces, and Douglas E. Cameron **[6]** has characterized maximal QHC-spaces and shown these to be submaximal. In Tychonoff spaces, lightly compact and pseudocompact are equivalent; and in Hausdorff spaces, QHC and H-closed are equivalent. We shall show that the maximal topologies of a class of topologies which include lightly compact and QHC are submaximal and T_1 spaces.

The topological space with topology τ on set X shall be denoted by (X,τ), the closure of a subset A of X with respect to τ is $\mathrm{cl}_\tau A$ and the interior of A with respect to τ is $\mathrm{int}_\tau A$, the complement of A with respect to X is $X - A$, the relative topology of τ on A is $\tau \mid A$, and the product of spaces (X_α, τ_α) for $\alpha \in \mathfrak{A}$ is $(\pi_{\mathfrak{A}} X_\alpha,\ \pi_{\mathfrak{A}} \tau_\alpha)$.

A topological space (X,τ) with property R is *maximal* R if whenever τ' is stronger than $\tau(\tau' \supset \tau)$, then (X,τ') does not have property R. In **[5]** it was shown that for a topological property R, (X,τ) is maximal R if and only if every continuous bijection from a space (Y,τ) with property R to (X,τ) is a homeomorphism. A topological space (X,τ) for which there exists a stronger maximal R topology is said to be *strongly* R. For $A \subseteq X$ the topology $\tau(A)$ with subbase $\tau \cup \{A\}$ is the *simple expansion* of τ by A.

We shall restrict our study to topological properties which satisfy some or all of the following:

P-1: contractive (preserved by continuous surjections)

P-2: regular closed hereditary

P-3: semi-regular (A topological property R is *semi-regular* if (X,τ) has property R if and only if (X,τ_S) has property R where τ_S is the semi-regularization of τ.)

P-4: contagious (A topological property R is *contagious* if