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WEAKLY ORTHOGONALLY ADDITIVE FUNCTIONALS, WHITE NOISE INTEGRALS AND LINEAR GAUSSIAN STOCHASTIC PROCESSES

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Let E be a Banach space. We prove a representation theorem for functionals on $L^2(E)$ that are additive on weakly orthogonal elements (as defind by Beck and Warren in Pacific J. Math. in 1972) and utilize the theorem to obtain a characterization theorem for linear Gaussian stochastic processes on $L^2(E)$ that take independent values on weakly orthogonal functions. This enables us give a new and very natural interpretation of the notion of weak orthogonality.

1. Introduction and notation. Investigation of orthogonally additive functionals (and operators) i.e., functionals on linear spaces that are additive on orthogonal (in a certain sense) elements, has a relatively long tradition. The first paper we could find was by Pinsker [11] who in 1938 gave the representation of orthogonally additive functionals on the Hilbert space equipped with the usual pythagorean ortnogonality relation. Then there was a series of papers by Friedman, Katz, Batt, Chacon, Sundaresan, Mizel, Drewnowski, Orlicz and Woyczyński (cf. e.g., [3], [9], [12] and references quoted therein) dealing with orthogonally additive functionals and operators on various, both concrete and abstract, vector lattices where orthogonality ment disjoitness (of supports). With the same orthogonality relation, orthogonally additive functionals on the Schwartz' spaces $\mathscr{D}$ were shown by Gelfand-Vilenkin [4] to be crucial in the theory of generalized stochastic processes with independent values at each point (the latter being of utmost importance in the quantum field theory) and that direction of research was later pursued for general linear processes in [12]. At last Gudder and Strawther ([6]) introduced an axiomatic notion of orthogonality in linear spaces that, in particular, included James orthogonality. In the same paper they also gave a representation formula for orthogonally additive functionals in that situation.

We remind that they call the relation $\perp$ in a real vector space V of dimension $\geqq 2$ an *orthogonality* if

(01) $x \perp 0, 0 \perp x$ for all $x \in V$;

(02) if $x \perp y$ and $x, y \neq 0$, then x, y are linearly independent;

(03) if $x \perp y$ then $ax \perp by$ for all $a, b \in \boldsymbol{R}$;

(04) if P is a two-dimensional subspace of V then for every $x \in P$ there exists $0 \neq y \in P$ such that $x \perp y$;