PACIFIC JOURNAL OF MATHEMATICS
Vol. 72, No. 1, 1977

WHEN IS A REPRESENTATION OF A BANACH *-ALGEBRA NAIMARK-RELATED TO A *-REPRESENTATION?

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Conditions are given which imply that a continuous Banach representation of a Banach *-algebra is Naimark-related to a *-representation of the algebra.

1. Introduction. The representation theory of a Banach algebra necessarily includes the notion of comparing representations to determine when they are essentially the same or related in important ways. Thus, if the algebra is a Banach *-algebra, then two *-representations are considered essentially the same if they are unitarily equivalent. When π is a representation of a Banach algebra on a Banach space X, we denote this Banach representation by the pair (π, X). A strong notion used to compare Banach representations is that of similarity.

DEFINITION. The Banach representations (π, X) and (φ, Y) of a Banach algebra A are similar if there exists a bicontinuous linear isomorphism V defined on X and mapping onto Y such that

$$\varphi(f)V = V\pi(f) \quad (f \in A) .$$

If (π, X) and (φ, Y) are similar, then the representation spaces X and Y are bicontinuously isomorphic. Thus the concept of similarity is limited to comparing representations that act on essentially the same Banach space. A notion that has proved useful in comparing representations that act on perhaps different representation spaces is that of Naimark-relatedness.

DEFINITION. Let (π, X) and (φ, Y) be Banach representations of a Banach algebra A. Then π and φ are Naimark-related if there exists a closed densely-defined one-to-one linear operator V defined on X with dense range in Y such that

(i) the domain of V is π-invariant, and

(ii) $\varphi(f)V\xi = V\pi(f)\xi$ for all $f \in A$ and all ξ in the domain of V.

The relation of being Naimark-related is in some ways a rather weak way of comparing representations. For this relation is not in general transitive [15, p. 242], and an irreducible representation can be Naimark-related to a reducible one [15, p. 243]. On the positive