PACIFIC JOURNAL OF MATHEMATICS
Vol. 74, No. 1, 1978

THE BOUNDEDNESS OF THE CANTOR-BENDIXSON ORDER OF SOME ANALYTIC SETS

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Let X and Y be complete separable metric spaces (Polish spaces). If E is a subset of $X \times Y$, and $x \in X$, then by the x-section of E, E_x, is meant $E \cap (\{x\} \times Y)$. By $P_S(E)$ is

$$P_S(E) = \{x: E_x \text{ is scattered}\} .$$

In this paper the following uniform boundedness principle for the Cantor-Bendixson order of analytic sets will be demonstrated.

THEOREM L. **Let W be an analytic subset of $X \times Y$ and let M be an analytic subset of X such that $M \subset P_S(W)$. Then there is a countable ordinal α such that the αth Cantor-Bendixson derived set of E_x is empty, for each x in M.**

Let us recall that if A is a subset of X, then the Cantor-Bendixson derivatives of A may be defined by transfinite induction as follows:

$$A^{(0)} = A$$

$A^{(\alpha)} = \bigcap_{\beta<\alpha} \{x: x$ is an acculumation point of $A^{(\beta)}$ and $x \in A\}$.

Recall that a subset, H, of a Polish space is scattered if, and only if H is a countable G_δ set, or equivalently, there is a countable ordinal γ such that the γth Cantor-Bendixson derived set, $H^{(\gamma)}$, of H is empty [5]. By the Cantor-Bendixson order of a subset H of a topological space is meant the first ordinal γ such that $H^{(\gamma)} = H^{(\gamma+1)}$. The Cantor-Bendixson order of every subset of a Polish space is necessarily less than ω_1 [5].

If $E \subset X \times Y$ and $M \subset X$, then E will be bounded on M provided there is an ordinal γ, $\gamma < \omega_1$, such that for each x in M, the Cantor-Bendixson order of E_x is $\leqq \gamma$; otherwise E will be said to be unbounded on M.

Let us note that in order to prove Theorem L it suffices to show that if E is an analytic subset of $X \times Y$ such that each x-section of E is scattered then E is bounded on the X projection of E, $\pi_X(E)$.

Theorem L has the following corollary:

COROLLARY 1. *Let X be an uncountable Polish space. Let $\mathscr{C}$ be any class of countable G_δ subsets of X which contains all the countable compact subsets of X except possibly countably many. Then no analytic set in X^2 can be universal for $\mathscr{C}$.*