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PRINCIPAL IDEAL AND NOETHERIAN GROUPS

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Let Π be a ring property. An additive group G is said to be an (associative) strongly Π-group if G is not nil, and if every (associative) ring R with additive group G such that R is not a zeroring has property Π. The (associative) strongly principal ideal groups, and the (associative) strongly Noetherian groups are classified for groups which are not torsion free. Some results are also obtained for the torsion free case.

(i) All groups considered here are abelian, with addition the group operation. Rings are not necessarily associative.

Let π be a ring property. A group G is said to be an (associative) π-group, denoted by $(A)\pi$-group, if there exists an (associative) ring R with additive group G such that R is not a zeroring, and R has property π. G is an (associative) strongly π-group, denoted by $(A)S\pi$-group. if G is an $(A)\pi$-group, and every (associative) ring with additive group G which is not the zeroring on G has property π.

If the only (associative) ring with additive group G is the zeroring, then G is said to be an (associative) nil group, denoted by (A) nil group.

The two ring properties π considered in this paper are:
1. every two-sided ideal is principal, denoted by *PI*,
2. every two-sided ideal is finitely generated, denoted by *N*.

In (ii) a complete characterization of the torsion $(A)SPI$ groups will be given. It will be shown that there are no mixed $(A)SPI$ groups. Some results concerning torsion free $(A)SPI$ groups will be obtained. In (iii), the torsion, and mixed SN groups will be completely characterized. Some results concerning torsion free SN groups will be given.

(ii) If X is a nonempty subset of a group or ring, (X) denotes the additive subgroup generated by X, and $\langle X\rangle$ denotes the ideal generated by X.

If $G = G_1 \oplus G_2$ is a group, π_{G_i} is the natural projection of G on G_i, for $i = 1, 2$.

LEMMA 1. *Let $G = H \oplus K, H \neq 0, K \neq 0$, be an ASPI-group. Then H and K are either both cyclic or both A nil.*