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THE SHEAF OF OUTER FUNCTIONS IN THE POLYDISC

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Let U^n be the unit polydisc in C^n. Define a presheaf by assigning to each relatively open subset W of $\bar{U}^n$ the multiplicative group of outer functions in the intersection $W \cap U^n$. If $\mathscr{O}$ denotes the associated sheaf, we prove that $H^q(\bar{U}^n, \mathscr{O})=0$ for all integers $q \geqq 1$.

1. **Introduction.** Classically, the outer functions in the open unit disc U are functions of the form

$$\lambda \exp \int_T \frac{w+z}{w-z} k(w) dm(w) ,$$

where m is the Haar measure on the unit circle T, k is an absolutely integrable real-valued function on T, and λ is a complex number of modulus one. Closely related to the class of outer functions is the Smirnov class $N^*(U)$, which consists of all functions that are holomorphic in U and admit an inner-outer factorization. The class $N^*(U)$ is an algebra, and the outer functions are precisely the invertible elements of this algebra. An alternative characterization of $N^*(U)$, considered by Rudin in [5], where it was extended to the polydisc U^n, is that a holomorphic function f in U belongs to $N^*(U)$ if and only if there exists a strongly convex function ϕ (depending on f) for which $\phi(\mathrm{Log}^+|f|)$ has a harmonic majorant. This definition can be extended naturally to arbitrary polydomains $W_1 \times W_2 \times \cdots \times W_n$, the requirement now being that $\phi(\mathrm{Log}^+|f|)$ have an n-harmonic majorant in $W_1 \times W_2 \times \cdots \times W_n$. We define the outer functions in $W_1 \times W_2 \times \cdots \times W_n$ to be the invertible elements of the algebra $N^*(W_1 \times W_2 \times \cdots \times W_n)$. (For the polydisc U^n, this definition can easily be seen to agree with the one given by Rudin in [5, Def. 4.4.3, p. 72].)

The correspondence that assigns to each polydomain W in C^n the group $O(W \cap U^n)$ of outer functions in the intersection $W \cap U^n$, defines a sheaf $\mathscr{O}$ on the closure $\bar{U}^n$ of U^n, which is locally determined in the sense that $\Gamma(\bar{U}^n, \mathscr{O})$ is canonically isomorphic to the group of outer functions in U^n. Our aim, in this article, is to show that the cohomology groups $H^q(\bar{U}^n, \mathscr{O})$ are trivial for all integers $q \geqq 1$.

Sheaves of a similar type (sheaves of germs of holomorphic functions satisfying boundary conditions on polydomains) have been studied by Nagel in [4], where a unified approach to many types of