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APPLICATIONS OF APPROXIMATION THEORY TO DIFFERENTIAL EQUATIONS WITH DEVIATING ARGUMENTS

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The authors utilize fundamental approximation theory concepts to establish an existence theorem for an initial value problem with a deviating argument. The techniques used to establish this theorem result in an algorithm to compute polynomial approximations to the solution of the initial value problem.

1. Introduction. In recent years several authors [2, 5, 8, 9, 10, 12] have considered best approximating in some sense the solutions to various types of nonlinear differential and integro-differential equations. These papers have generally been devoted to examining questions involving the existence of best approximations and with proving that appropriate sequences of approximations converge to a solution of the differential or integro-differential equation. The best approximation problems of the above references are ordinarily nonlinear and consequently computational questions are not readily resolved.

More recently Allinger [1], Henry [1, 6] and Wiggins [6] have developed alternative approximation theory approaches that are computationally adaptive.

In this regard [6] deals with nonlinear initial value problems without deviating arguments, whereas [1] primarily considers linear initial value problems with deviating arguments. The approximation problems of [6] and [1] are basically different. The goal of the present paper is to extend the results of [6] to arbitrary order and to the more difficult deviating argument case. The theory developed in the present paper will not contain the linear theory of [1]; in a subsequent paper [7] the authors will extend the results in [1] and relate those extensions to the fundamental results of the present paper.

2. The initial value problem. Consider the scalar initial value problem

$$\begin{aligned}(1)\quad & x^{(n)}(t) - \hat{f}(t, x(t), \cdots, x^{(n-1)}(t), x(h(t)), \cdots, x^{(n-1)}(h(t))) = 0\ ,\\ & x^{(i)}(0) = \alpha_i,\ i = 0, 1, \cdots, n-1,\ t \in J = [-\gamma, \tau]\ ,\end{aligned}$$

where $\gamma \geqq 0$, $\tau \geqq 0$, and $\gamma + \tau > 0$. This initial value problem is